Antonine Physics AS

Topic 4 DC Electricity

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

Electricity is an important way of transferring energy. It has the advantage that it can be distributed nationwide from central power stations, where it is generated in bulk. Electrical phenomena have been recognised by physicists for about two hundred years and have been put to work over the last one hundred and fifty years.

In this unit in the first year (AS), you will study the physics behind DC electric circuits.

There is some more challenging material that is NOT expected at 1st Year. Do NOT attempt to do these sections unless you are feeling confident enough!

Alternating Currents will be studied in the 2nd Year (A-level).

Topic 4 Electricity

Tutorial 4.01 Basic Electrical Measurements

All Syllabi

Contents

4.011 Measurement

The instruments that are of most use to the physicist and the electrical engineer are the **voltmeter** and **ammeter**. With these, we can directly measure:

- **Potential Difference** (or voltage).
- **Current**.

We can then use the data to get:

- **Resistance** (volts ÷ amps).
- **Power** (volts × amps).

The **multimeter** is often used, as it can measure voltage and current (but not at the same time).

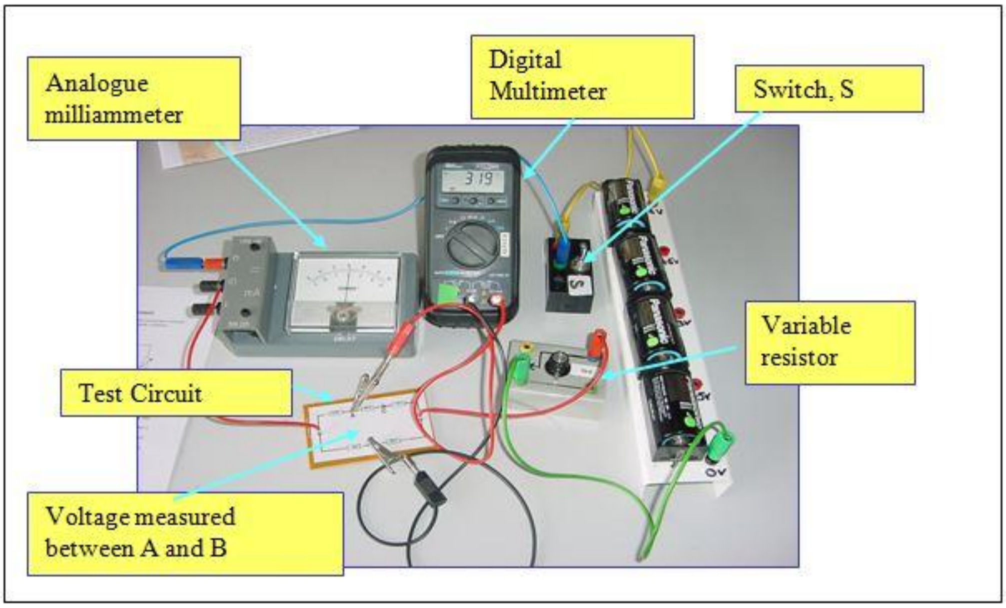


Figure 1 Using meters for simple electrical measurement

Notice in *Figure 1* that there is an **analogue** meter (a meter with a scale) and a **digital** meter (a meter where the read-out is a number). It is important that you learn how to use these instruments correctly in your practical work. If in doubt, ask your teacher.

Current is measured with an ammeter, which is wired in series with the component. The voltmeter is wired in parallel with the component.

4.012 Circuit Diagrams

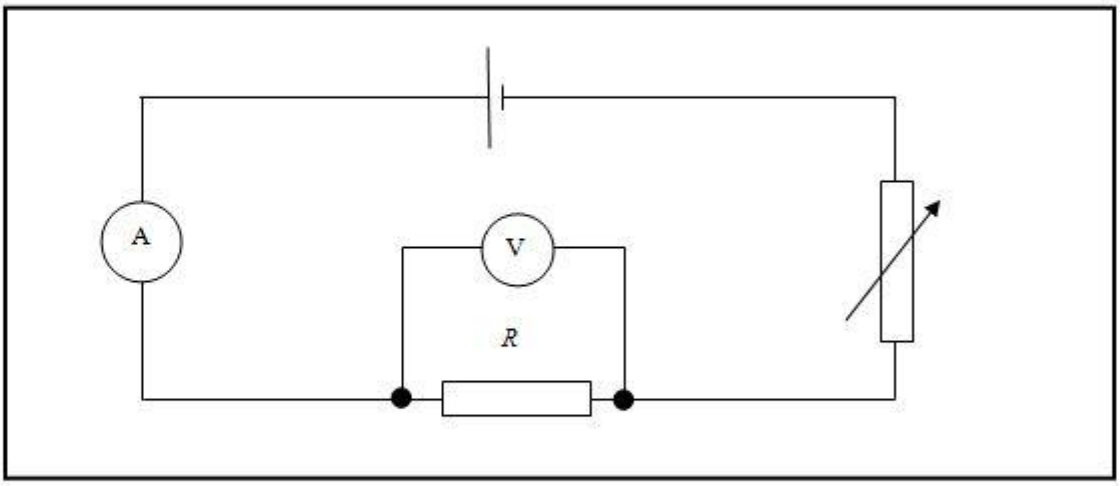


Figure 2 A circuit diagram for a simple electrical circuit.

The diagram (*Figure 2*) above is a **circuit diagram**. It shows how the components are connected together. It uses standard symbols which all electrical engineers will understand. You do not have to be an artist to draw one, but follow these simple rules:

- Use a pencil and a ruler.
- Draw the lines horizontally and vertically.
- Ensure that all the components are there, including the power supply.
- Use the correct symbols.
- Where there are junctions (wires joined together), there is a black circle, which represents a blob of solder.

You knew that anyway, didn't you?

Figures 3 and *4* show a range of components and their standard symbols. Sometimes other symbols are used.

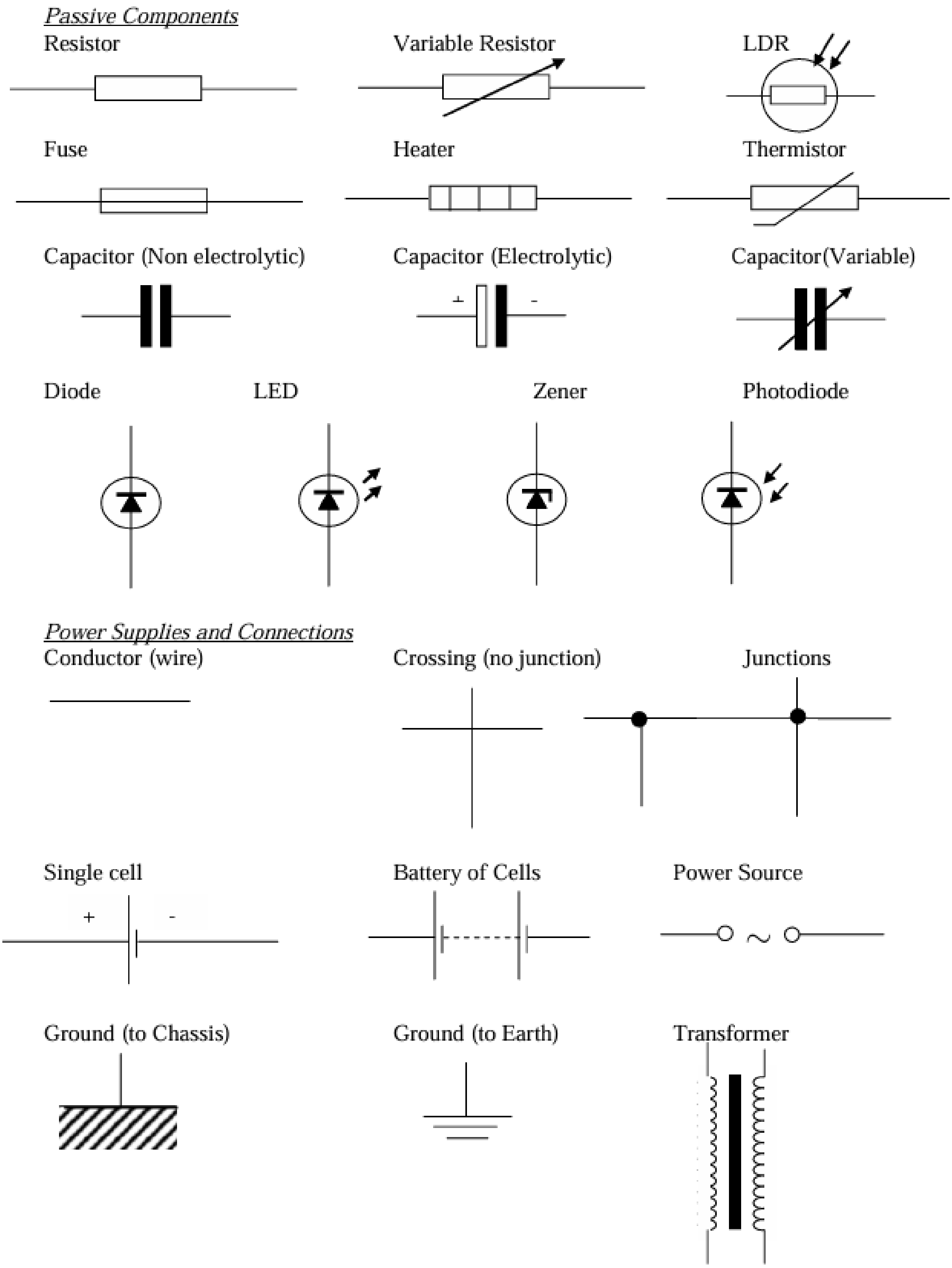


Figure 3 Component Symbols

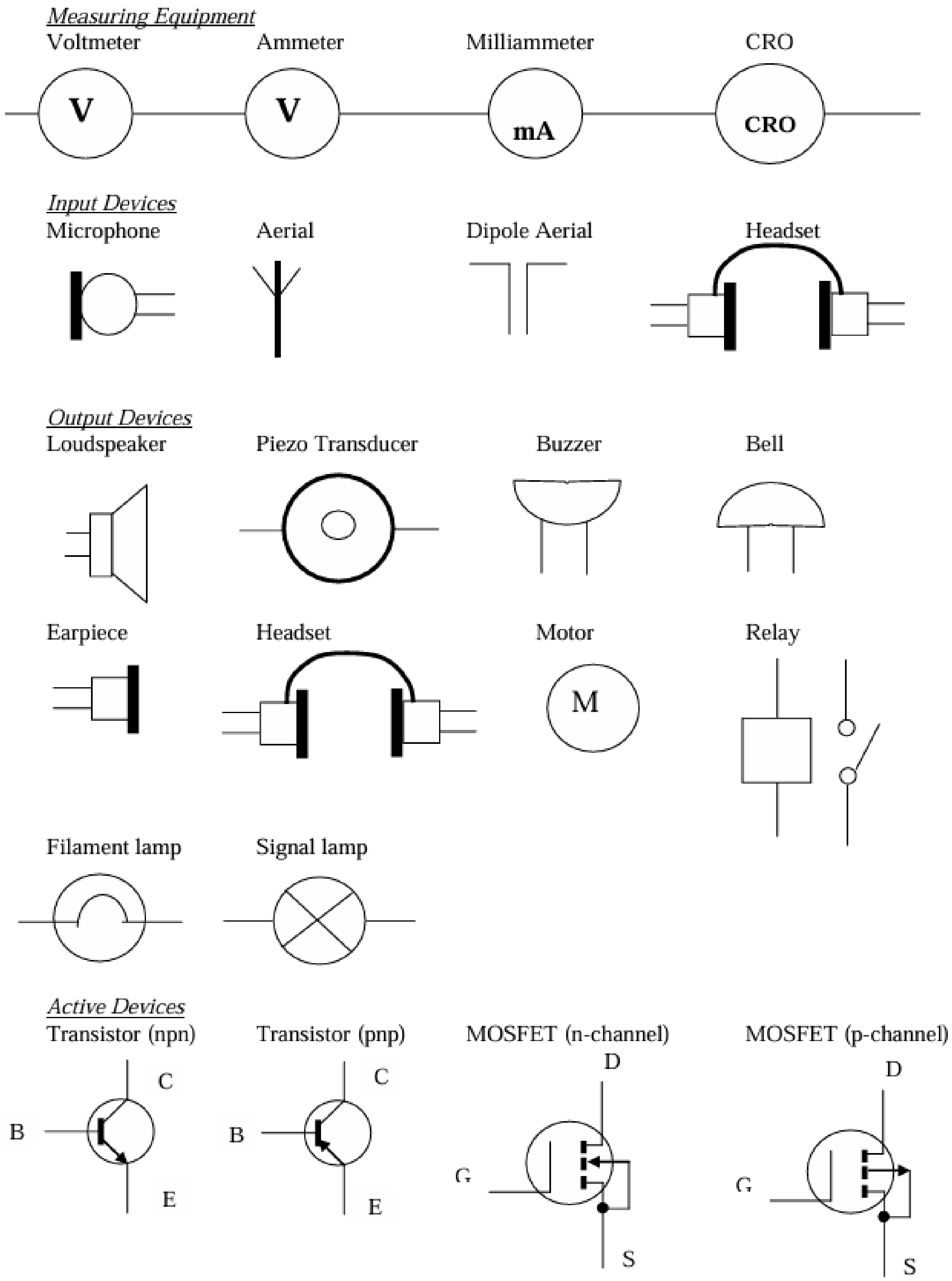


Figure 4 More component symbols

In word-processed documents, use graphics to produce a circuit diagram if you can.

4.013 Current and Charge

The base electrical quantity is **current**, the flow of charge. All other electrical quantities are derived from it. Current is measured in **ampères**, or **amps** (A).

Charge is measured in **coulombs** (C), which is defined as:

1 coulomb is the quantity of charge carried past a given point if a steady current of 1 amp flows for 1 second.

$$1\ \text{C} = 1\ \text{A s}^{-1}$$

A single electron carries a charge of **1.6×10^{-19} C**.

1 coulomb is equivalent to 6.2×10^{18} electrons. It is much more convenient to use this rather than counting individual electrons. You would buy a 1 kg bag of sugar rather than counting all the crystals in it.

All charges have a **whole number multiples** of the electronic charge, e. We say that charge is **quantised**.

Charge and current are linked by a simple formula:

Charge (C) = current (A) × time (s)

$$Q = It \quad \text{................ } \textit{Equation 1}$$

In the syllabus the formula is written in physics code as:

$$I = \frac{\Delta Q}{Dt} \quad \text{............... } \textit{Equation 2}$$

The symbol Δ is Delta, a Greek capital letter 'D', meaning "change in".

There are some important **multipliers** for current:

- 1 microamp (1 mA) = 1×10^{-6} A
- 1 milliamp (mA) = 1×10^{-3} A

These are useful when we are dealing with small currents. However, we must remember to convert to **SI units** for doing calculations. Watch out for this bear trap!

4.014 Potential Difference

Potential Difference is defined as **energy per unit charge transferred to useful energy by the circuit**. Electromotive force (E.m.f.) is the energy per unit charge supplied by the source to the circuit.

The unit of potential difference is the **volt** (V). Using the definition, we can define the volt as **Joules per Coulomb**.

$$1\text{ V} = 1\text{ JC}^{-1}.$$

$$\text{Potential difference (V)} = \frac{\text{energy converted (J)}}{\text{Charge (C)}}$$

In physics code we write:

$$V = \frac{\Delta E}{\Delta Q}$$ .... *Equation 3*

Potential difference is often referred to as **voltage**. It will be often referred to as such in these notes. In the exam, the questions refer to potential difference, but you will NOT lose marks for using "voltage" in your answers.

Conventional current goes from **positive** to **negative**. **Electrons** carry energy around the circuit; they go from **negative** to **positive**. In the early days, physicists didn't know about the electron, which is why they got it all wrong. Correction would require a complex re-write of the Laws of Physics, a task which no-one is likely to be bothered to tackle. So, all conventional currents are from positive to negative. All currents are treated as **conventional**. See *Figure 5*.

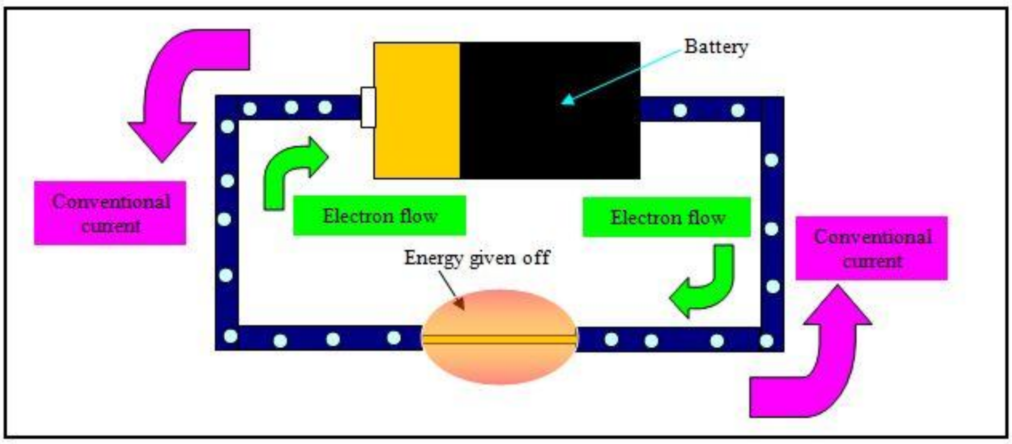


Figure 5 Conventional current flows from positive to negative

4.015 Sources of Voltage

Examples of **sources of voltage** include:

- Car batteries.
- Dry cells.
- Rechargeable cells.
- Mains.
- Generators.

A single **cell** normally provides about 1.5 volts. A lab-pack might provide 15 V, and a generator can provide up to 25 000 V. Without a **voltage source**, current cannot flow.

If we connect the cells in series, we have a **battery** of cells. The battery below has four cells in **series**. Its voltage is 4 × 1.5 = 6 V. (*Figure 6*)

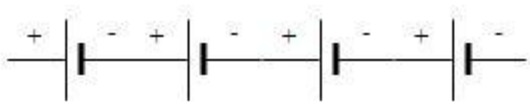


Figure 6 A battery of series cells

We can wire cells in **parallel** as well. This enables us to get a higher current. A minibus might have two 12 V batteries in parallel to provide the massive current (1000 A) taken by the starter motor for its diesel engine. (*Figure 7*)

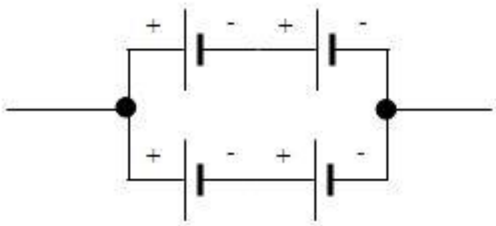


Figure 7Wiring four cells as a battery of parallel cells

This combination of cells will give us an output voltage of 3 V, not 6 V. We would be able to get twice the current from this set of cells.

The combination of the cells shown below gives out a potential difference of 4.5 V:

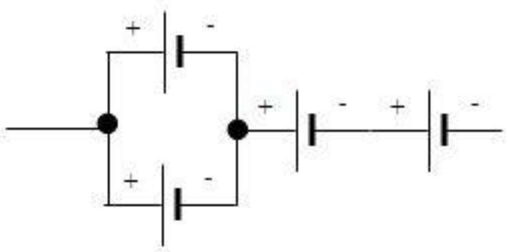


Figure 8 Cells in series and parallel

This is because the voltage of the parallel combination is 1.5 V

The two cells in the parallel combination will last twice as long, since the current will be half that taken from the two series cells.

Batteries are often rated in **amp-hours**. A 1-amp-hour battery can give out a current of 1 amp for 1 hour. The charge contained is:

$$Q = 1\ \text{A} \times 3600\ \text{s} = 3600\ \text{C}$$

When a cell is **discharged**, its **terminal voltage** varies as shown in the sketch graph (*Figure 9*).

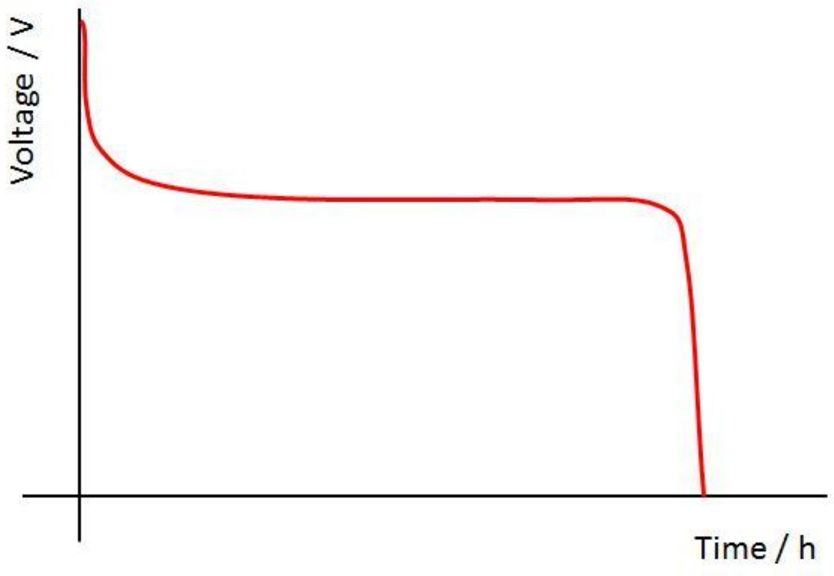


Figure 9 Graph showing voltage fall-off during discharge

There is a rapid fall-off of voltage when the cell is first connected to the circuit. Then the voltage remains steady for a long period of time. Then it drops of very rapidly as the reactants are used up, leading the battery to go **flat**. You can check this out by connecting a fresh cell to a resistor and using a data-logger to record the voltage. You will need a data-logger as the time taken is quite long, longer than your physics lesson.

4.016 Primary and Secondary Cells

A **primary cell** is one that is used once and is disposed of when it's flat. The electrochemical reaction is **irreversible**, so the cell **cannot be recharged**. Although we might think that replacing primary cells is expensive, there is a definite advantage, which is that the primary cell loss its charge at a very low rate when stored. This is about 8 % a year. Some manufacturers claim that primary cells can be recharged, and chargers are available to do this. However, the reactants are not restored to their original states or location. The performance and life of recharged primary cells is significantly below what would be expected from a fresh cell.

Secondary cells are rechargeable. The electrochemical reactions are reversible with the reactants being able to be restored to their original state. However, the capacity tends to reduce as the cell goes through more **charge and discharge cycles**. Therefore, the number of charge and discharge cycles is limited to about 1000 for low-capacity batteries. For high-capacity batteries, the number is about 500 times. When such cells are stored, there can be a leakage current that results in the cells going flat after a period. Users of a cordless drill may find that they have to charge the batteries before they can start the job. A NiCd battery can lose up to 10 % of its charge in the first 24 hours, then about 10 % a month.

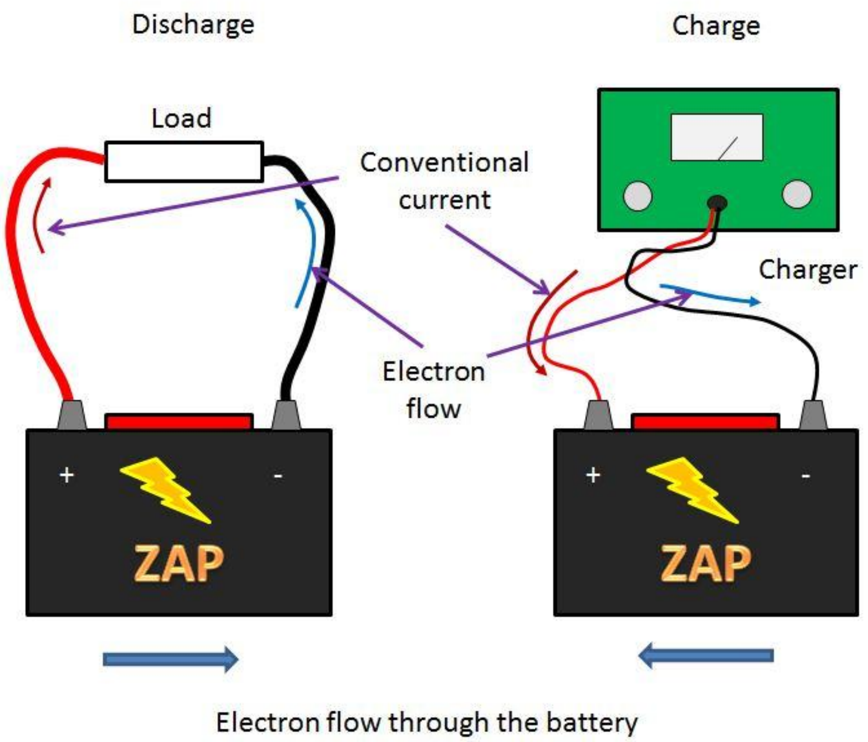


Figure 10 Electron flow through a battery during discharge and charging.

When the battery is connected to the load, the electrons flow from the **negative** terminal, through the load, to the **positive** terminal (*Figure 10*). This is the **opposite direction** to the **conventional current**. The electron flow will last until the reaction is completed.

The **charger** has its **positive** terminal connected the battery **positive**, and its **negative** to the battery **negative**. The electrons are pumped in by the charger, and their flow is in the **opposite** direction to their normal flow in the battery. This means that the electrons go **up the potential hill** to the top. The electrons leave through the positive terminal and go back to the charger. In this case, the positive terminal of the battery is a source of electrons.

Secondary cells can give out a very large current when needed. A car battery can easily produce 500 A.

Other problems include:

- Memory effects.
- Damage due to overcharging.
- Damage due to too rapid a charge.
- Loss of efficiency when the cell is cold.
- Accidental damage to such a cell can be very dangerous, causing a fire.

Examples include:

- Lead Acid batteries for cars.
- Nickel cadmium (NiCd).
- Nickel metal hydride (NiMH).
- Lithium Ion (Li-ion).

Lithium-ion cells are light and have a good energy density. Therefore, they can give out the high currents needed for a drone to fly for an extended time.

Large Li-ion batteries are used to power modern electric cars. They are **heavy**. Even though electric motors are much more efficient than petrol or diesel engines, even the largest battery has about as much **energy** as 15 litres of petrol. Therefore, the **range** between charges is limited, about 450 km. They need specialised **charging points** to charge up the batteries. So-called "**rapid charge**" points (if available at all) take 20 – 30 minutes to place a reasonable amount of energy, whereas it takes no more than five minutes to fill up a tank of petrol and pay for it. You may well have to wait for someone else to finish their charge, so you need to be **patient** (I am not!). You can have a charging point at home, but you need to have a **driveway** to park your EV while charging. This will also take a long time. Smaller EVs can be plugged into a **wall socket** using a special lead, but the charge time is very much longer.

You could find that you have an extended answer question on this in the exam, so take note of these points.

4.017 Resistance

Resistance in a wire is the opposition of a wire to the flow of electricity. It is caused by collisions between the electrons and the atoms in the wire. The hotter the wire, the more chance there is of a collision. Therefore, hot wires have more resistance. The formula for resistance is:

$$\text{Resistance (ohms)} = \frac{\text{potential difference (volts)}}{\text{current (amps)}}$$

In physics code we write this as:

$$R = \frac{V}{I} \quad \text{.......... } \textit{Equation 4}$$

Or more commonly:

$$V = IR \quad \text{.......... } \textit{Equation 5}$$

The unit for resistance is ohm (Ω). (The curious symbol 'Ω' is *Omega*, a Greek capital letter long Ō.)

An alternative unit for ohms is **volts per ampère**:

$$1\ \Omega = 1\ \text{V A}^{-1}$$

Watch out for these bear traps in electrical calculations:

- Time must be in seconds
- Make sure you convert milliamps to amps

4.018 Measuring Electrical Quantities

We can measure voltage and current using a circuit like this. You will be familiar with this from GCSE, and you will be expected to set it up as a matter of course (*Figure 11*).

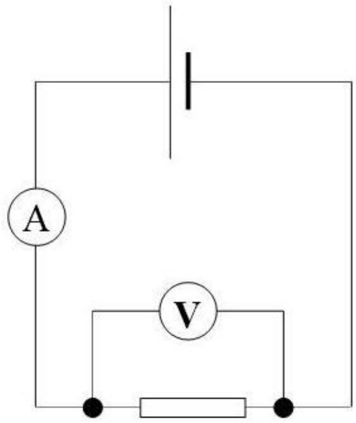


Figure 11 Simple Electrical Measurement

The **voltmeter** is wired in parallel with the component, and the **ammeter** is wired in series with the components.

Normally we treat the instruments as **perfect**. This means that:

- An ammeter has zero resistance.
- A voltmeter has infinite resistance.

In reality an ammeter has a very low, but measurable resistance. Normally we ignore this.

Analogue voltmeters have a high resistance. Normally this has little effect, but if we are measuring high value resistances, the current taken by the voltmeter can alter the result.

A digital voltmeter has a very high resistance indeed and can be regarded as perfect.

4.019 Voltmeters as Readouts

A voltmeter can be **calibrated** to give a reading that is not a voltage. For example, an electronic thermometer gives out a variable voltage, but this is shown as a **temperature**. The picture below shows a voltmeter calibrated as a photographer's light meter (*Figure 12*).

Figure 12 An analogue photographer's light meter. Image by El Grafo, Wikimedia Commons

At a certain light level, the electronic circuitry gives out a particular voltage. The photographer doesn't need to know what that voltage is. Instead, she needs to interpret the output into the shutter exposure and the aperture of the lens. The calibration is done by the manufacturer.

A more modern device has a **digital output** (*Figure 13*).

Figure 13 A digital light meter

The output is a direct readout, so no skills interpreting the meter are needed.

4.020 Meters of known resistance (A-level and IB)

This kind of problem is likely to appear in A-level papers rather than AS-level. It is quite involved and challenging, so make sure you understand each step and don't rush it. You are NOT expected to know this for the 1st Year (AS) exams.

An **analogue** meter can be made to measure voltage or current by adding an external resistor:

- An ammeter uses a **shunt**, which is a low value resistor connected in parallel.
- A voltmeter uses a **multiplier**, which is a high value resistor connected in series.

You can't have the two on at the same time! A typical meter found in a school laboratory is shown below (*Figure 14*).

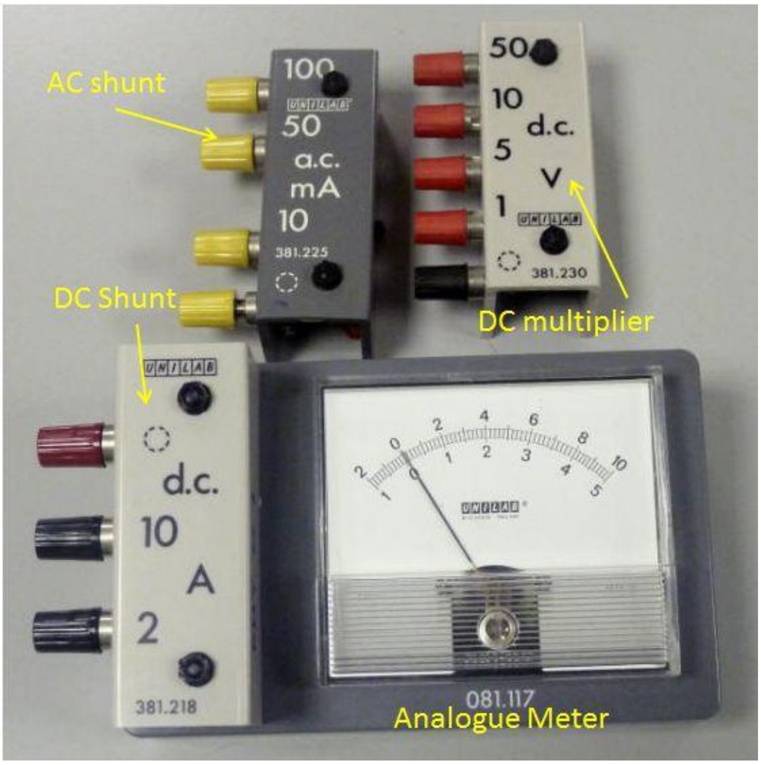


Figure 14 Shunts and multipliers on an analogue mater.

Notice that there are separate shunts for AC and DC. There is also a separate multiplier for AC voltage.

Figure 15 shows the shunt resistors and the multiplier resistors in an analogue **multimeter**:

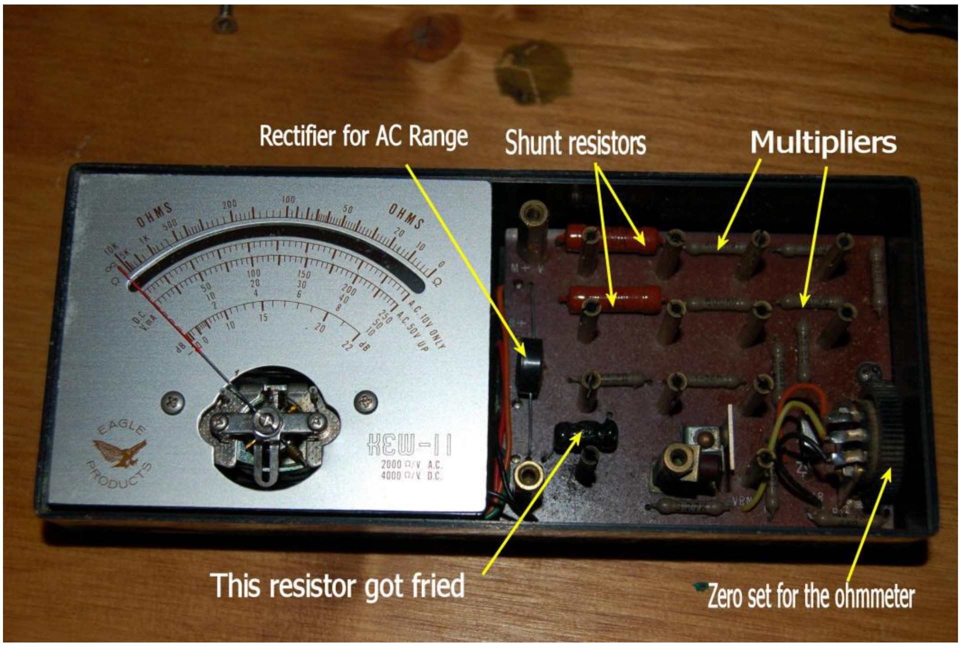


Figure 15 Shunts and multipliers in an analogue multimeter.

The same applies to digital multimeters, but the shunts and multipliers are not so easy to identify.

The real **voltmeter** has a circuit diagram like this (*Figure 16*).

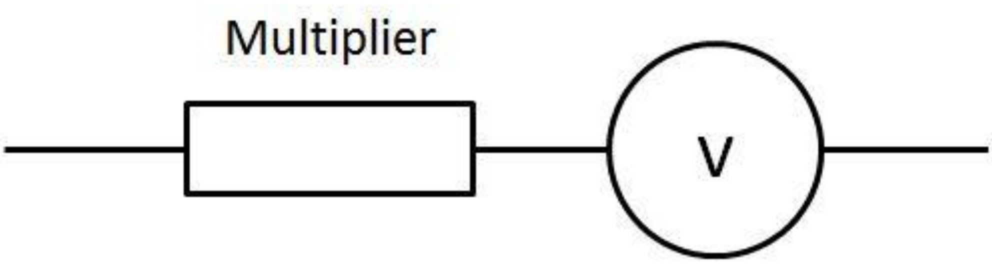


Figure 16 Multiplier for a voltmeter.

A **perfect** voltmeter has an **infinite** resistance. A digital voltmeter has a very high resistance and can be considered to be almost perfect. An analogue voltmeter will have a resistance of about 50 kilohms. We **model** the voltmeter of known resistance as a **perfect voltmeter in parallel with a known resistance** (*Figure 17*).

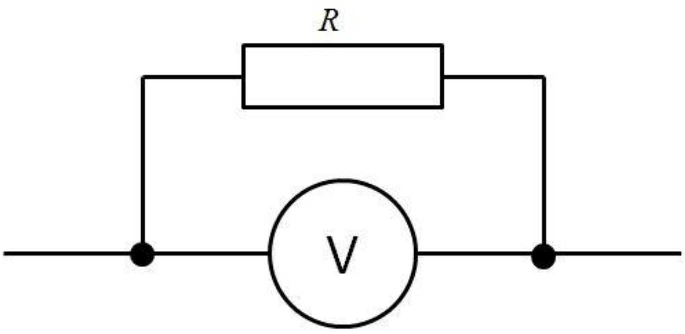


Figure 17 Modelling a real voltmeter as a perfect voltmeter in parallel with a resistor or resistance R.

The circuit diagram shows how the meter is MODELLED.

The multiplier is actually wired in series.

Before you look at this worked example, make sure you are confident in the concept of a cell with **internal resistance**.

Worked example

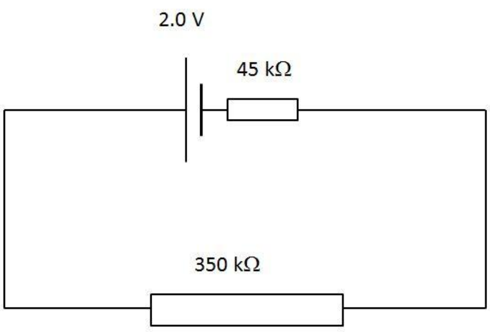
A voltmeter of resistance 35 kΩ is used to measure the voltage across a 350 kΩ resistor. The source providing the current can be modelled as a voltage source of 2.0 V with an internal resistance of 45 kΩ.

(a) What is the voltage across the 350 kΩ resistor when the voltmeter is NOT connected?

(b) What is the voltage that is actually read from the meter?

Answer

We model the circuit like this.



(a) We need to find the total resistance of the circuit:

$$R = 45\ 000\ \Omega + 350\ 000\ \Omega = 395\ 000\ \Omega$$

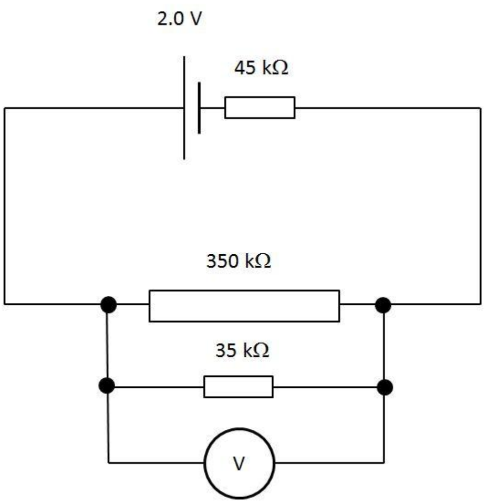
Work out the current:

$$I = 2.0\text{ V} \div 395\ 000\ \Omega = 5.06 \times 10^{-6}\text{ A}$$

Now work out the voltage across the 350 kΩ resistor:

$$V = 5.06 \times 10^{-6}\text{ A} \times 350\ 000\ \Omega = 1.77\text{ V} = \mathbf{1.8\ V}\ (2\text{ s.f.})$$

(b) A voltmeter with a known resistance is treated as a **perfect voltmeter in parallel with a known resistor**. With the voltmeter added our circuit becomes:

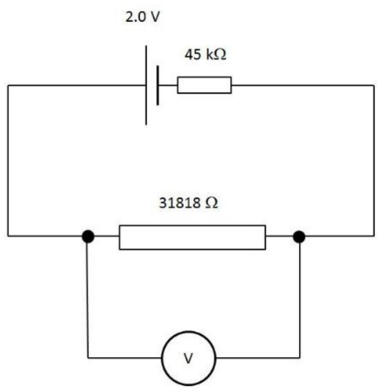


Now we need to work out the parallel combination of resistors:

$$R^{-1} = (350\,000\ \Omega)^{-1} + (35\,000\ \Omega)^{-1} = 31.4 \times 10^{-6}\ \Omega^{-1}$$

$$R = 31818\ \Omega$$

Now our circuit with the perfect voltmeter becomes (*Figure 20*):



Now we work out the total resistance:

$$R = 45\,000\ \Omega + 31818\ \Omega = 76818\ \Omega$$

Then we can work out the current:

$$I = 2.0\ \text{V} \div 76818\ \Omega = 26.04 \times 10^{-6}\ \text{A}$$

Finally, we work out the voltage read by the meter:

$$V = 26.04 \times 10^{-6}\ \text{A} \times 31818\ \Omega = 0.828\ \text{V} = \mathbf{0.83\ V}\ (2\ \text{s.f.})$$

Note that the rounding appropriate significant figures is done at the last step.

Note that the last example needed a lot of steps.

A perfect **ammeter** has zero resistance. While we can get a very nearly perfect voltmeter with a digital instrument, the same does not apply to an ammeter. There is always a small resistance to the ammeter. A real ammeter is a meter in parallel with a low value resistor called a **shunt**. You will sometimes find in engineering text-books references to motors being shunt-wound. This simply means that the rotor coils and the field coils are wired in parallel.

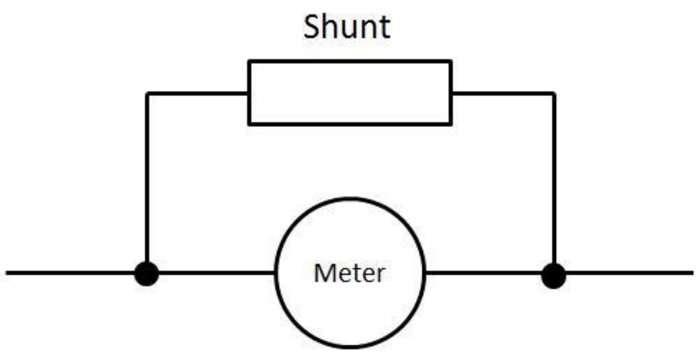


Figure 18 A shunt is a resistor in parallel.

The meter is reading the voltage across the shunt resistance.

We model the ammeter as a **perfect ammeter** (i.e. one with zero resistance) **in series with a resistor of known resistance**, like this (*Figure 19*):

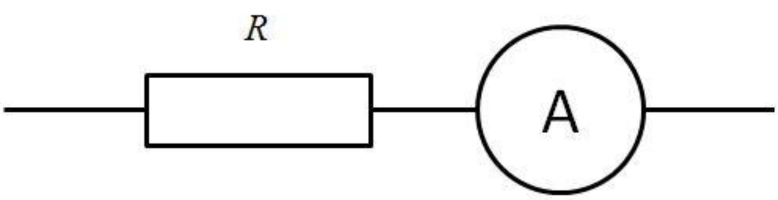


Figure 19 Modelling an ammeter as a perfect ammeter in series with a resistor of resistance R

Using low values of resistance, there is not a great deal of difference compared to using ideal meters. However, with very high value resistors, as shown in the worked example, the differences become more obvious.

A multimeter can also be used as an **ohmmeter**. In this case, the meter has a battery, and it measures the current. The scale has its zero point when the maximum current is shown. When zero current flows, the resistance is **infinity**.

Figure 20 An analogue multimeter can be used as an ohmmeter

The limitation is for high resistances. Try measuring 7600 ohms with this scale.

Tutorial 4.01 Questions

4.01.1

What do you think an electron is?

4.01.2

Show that 1 coulomb is 6.2×10^{18} electrons.

4.01.3

A charge of 1.24 C flows in a period of 0.63 s. What is the current?

4.01.4

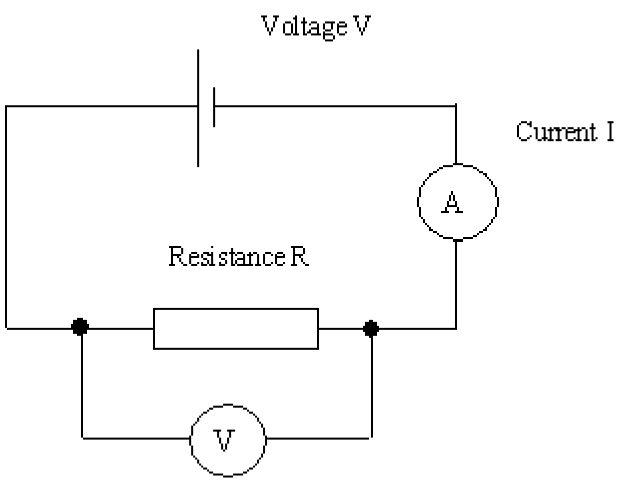
A cordless drill operates using a 14.4 V battery pack. The battery is rated at 2.0-amp hours which means that it can deliver a current of 2.0 amps for a period of 1 hour. How much energy is held by the battery?

4.01.5

What do you understand by the term resistance?

4.01.6

Use the circuit below to complete the table:

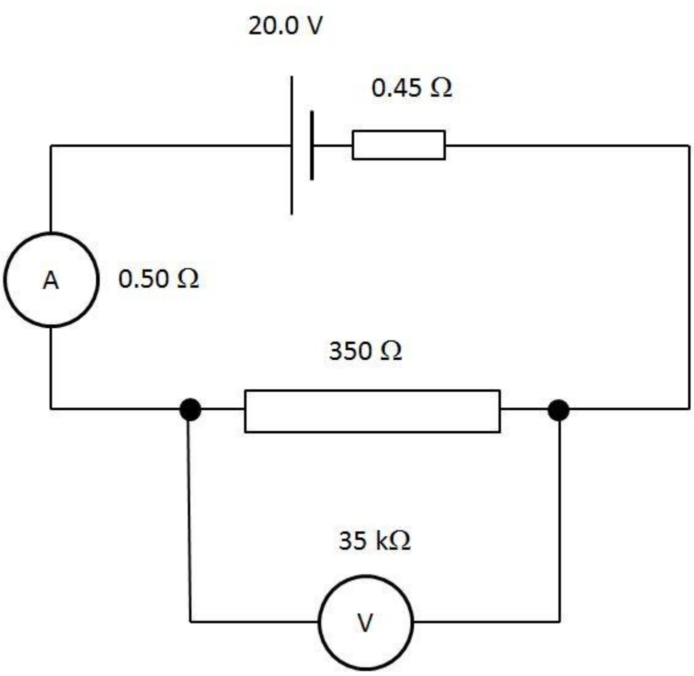


	Voltage, V	***Current, I***	***Resistance, R***
(a)		0.30 A	18 Ω
(b)	12 V		88 Ω
(c)	14.4 V	0.52 A	

4.01.7

(Challenge - Revise internal resistances of cells before trying this one out)

The circuit shows a battery of EMF 20.0 V that has an internal resistance of 0.45 Ω. It is connected to a 350 Ω resistor. The potential difference is measured using a voltmeter of resistance 35 kW, while the current is measured using an ammeter of resistance 0.50 W. The circuit is shown in the diagram below:



(a) Show that the current from the cell is about 58 mA.

(b) Calculate the reading of the voltmeter, which is known to be accurately calibrated.

Tutorial 4.02 Ohm's Law

All Syllabi

Contents

4.021 Ohm's Law

Resistance is the opposition to the flow of an electric current. Resistance in a conductor is thought to arise due to the **collisions** between the **charge carriers** and the **ions** in the lattice. The internal energy rises, so the conductor gets hot. The hotter the conductor, the greater the probability of a collision between an ion and an electron. The resistance in hot conductors rises.

Resistance is also the **ratio of the voltage to the current**, described in the simple equation $R = V/I$. In a metallic conductor, we find that if we alter the voltage or the current, the other variable changes in such a way that the ratio remains **constant**.

Equation:

$$R = \frac{V}{I}$$

Resistance (Ω) → R; Potential difference (V) → V; Current (A) → I

Equation 6

This is **Ohm's Law**, which states:

The current in a metallic conductor is directly proportional to the potential difference between its ends provided that the temperature and other physical conditions are the same.

A conductor that obeys Ohm's Law is called an **ohmic** conductor.

However, if the temperature does not stay the same, Ohm's Law does not apply, and it is easy to end up with anomalous results. When doing experiments on Ohm's Law, it is a good idea to turn the power supply off because it:

- Stops the batteries from running down.
- Prevents the resistor getting hot and increasing its resistance.

Ohm's Law applies to metallic conductors and carbon. It does not apply to conducting materials like liquid electrolytes, for example a solution of sodium chloride. Semi-conductors also show different behaviour, as we will see later.

4.022 Conductance

Conductance is the reciprocal of resistance. It's a term often used by electrical engineers in preference to resistance. Conductance is given the physics code *G* and the units Siemens (S). It is related to resistance by the equation:

$$G = \frac{1}{R}$$

.... *Equation 7*

So, in combining *Equations 3* and *4*, it doesn't take a genius to see that:

$$G = \frac{I}{V}$$

.... *Equation 8*

A graph of voltage (y-axis) against current (x-axis) gives a gradient of **resistance**. A graph of current against voltage give **conductance** (*Figure 21*).

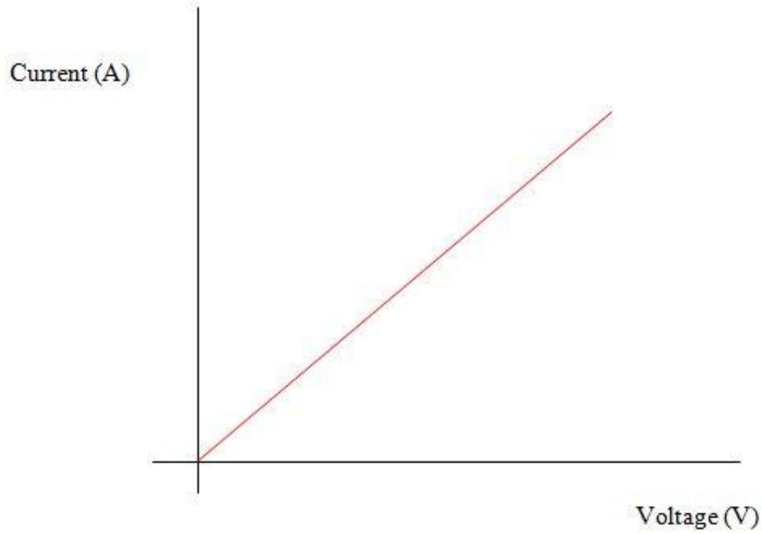


Figure 21 Graph of current against voltage.

Make sure you read carefully the axes on the graph.

In the exam, they could be either way round.

4.023 Reactance (A-level only)

Certain components that are used with **Alternating currents** (AC) have a kind of resistance called **reactance**. We will use the definition of resistance as:

The ratio of the potential difference to the current.

We find that that the **reactance** is the **ratio of the RMS voltage to the RMS current**. We will study this in more detail in the AC Theory tutorials.

For the reactance of a **capacitor**, physics code X_C, we can write:

$$X_C = \frac{V_{RMS}}{I_{RMS}}$$

.... *Equation 9*

The units for reactance are **Ohm** (Ω), just like resistance.

Similarly for the reactance of an **inductor**, X_L, we can write:

$$X_L = \frac{V_{RMS}}{I_{RMS}}$$

.... *Equation 10*

4.024 Changing Physical Conditions (Extension)

When we defined Ohms Law for a metallic conductor, we had a proviso that the **temperature and other physical conditions stayed the same**. What happens if they don't remain the same? The simple answer is that the **resistance** changes. The most obvious ways of changing the physical conditions are:

- Heating the wire up.
- Stretching the wire.

If the wire gets hot, the internal energy increases so that the chances of a collision between an ion and a proton are greater. Therefore, the resistance of the wire increases.

If we stretch the wire, which has a fixed volume, the length will increase. Therefore, the width will decrease. Therefore, the resistance will increase. We will think about this case more with resistivity.

The way the resistance of a wire changes with temperature depends on:

- the change in temperature.
- the material of the wire itself.

When we refer to a resistance of a resistor, the convention is that the resistance is assumed to be measured at 20 °C, although some authorities use 0 °C. You don't need to know this for A-level. The change in resistance at other temperatures is given by the formula:

$$R = R_0(1 + \alpha\Delta\theta)$$

.... Equation 11

The terms are:

- R - resistance (Ω);
- R_0 - reference resistance (Ω).
- α - **temperature coefficient** of resistance for the conductor ($^\circ C^{-1}$).
- $\Delta\theta$ - the temperature **change** ($^\circ C$).

The temperature change is the **difference** between the **quoted temperature** (θ) and the **reference temperature** (θ_0). It does not matter if you quote temperatures in Kelvin (K) or Celsius; it's the difference that matters.

Here are some values for the temperature coefficients:

Material	***Element/Alloy***	***a /°C^{-1}***
Nickel	E	0.005866
Iron	E	0.005671
Molybdenum	E	0.004579
Tungsten	E	0.004403
Aluminium	E	0.004308
Copper	E	0.004041
Silver	E	0.003819
Platinum	E	0.003729
Gold	E	0.003715
Zinc	E	0.003847
Steel	A	0.003000
Nichrome	A	0.00017
Nichrome V	A	0.00013
Manganin	A	0.000015
Constantan	A	0.000074

Source: https://www.allaboutcircuits.com/textbook/direct-current/chpt-12/temperature-coefficient-resistance/

Constantan makes a particularly good resistance wire as the temperature coefficient is very low. Therefore, the resistance remains constant (to 2 s.f.) over a wide range of temperatures.

Worked Example

A resistor has a value of 15 Ω at a temperature of 20°C. The resistor is made of iron wire which has a temperature coefficient (α) of 0.005671 °C^{-1}.

What is the resistance if the temperature is raised to 400 °C?

Answer

Temperature change:

$\Delta\theta$ = 400 °C - 20 °C = 380 °C

Use:

$$R = R_0(1 + \alpha\Delta\theta)$$

Substitute:

R = 15 Ω × [1 + (0.005671 °C^{-1} × 380 °C)] = 47.3 Ω

You can see that the resistance increases about three times.

The graph below (*Figure 22*) shows the change in resistance with temperature:

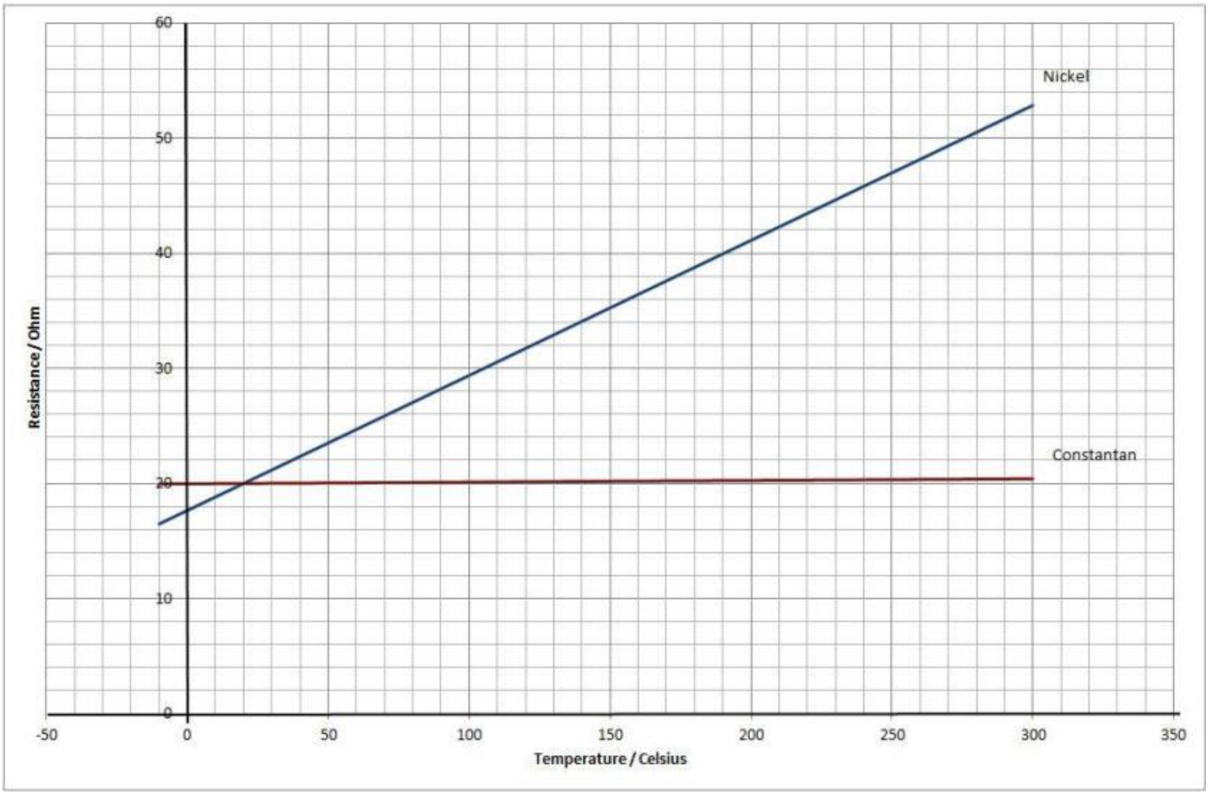


Figure 22 How resistance changes with temperature.

The graph is linear, but NOT directly proportional. The **gradient** gives a value for the temperature coefficient.

The **reference resistance** is 20 Ω. The graph shows the extremes on the list, **Nickel** (α = 0.005866 $^{o}C^{-1}$) and **Constantan** (α = 0.000074 $^{o}C^{-1}$). You can see that the change in Nickel is quite substantial, with an increase in resistance of about 3 times. For constantan, the change is insignificant - which is why the alloy is called constantan. Its resistance is constant. The limit for the temperature range is from -273 ^{o}C (0 K) to the melting point of the metal.

Tutorial 4.02 Questions

4.02.1

What are the key points to Ohm's Law?

4.02.2

A component takes a current of 0.35 A from a 12 V supply. What is the resistance of the component?

4.02.3

And what is its conductance?

Tutorial 4.03 Voltage-Current Characteristics

All Syllabi

Contents

4.031 Measuring Voltage against Current

We can easily measure voltage and current, using the data to plot **voltage current graphs**. We use the following circuit, which you probably did in Year 10 (*Figure 23*).

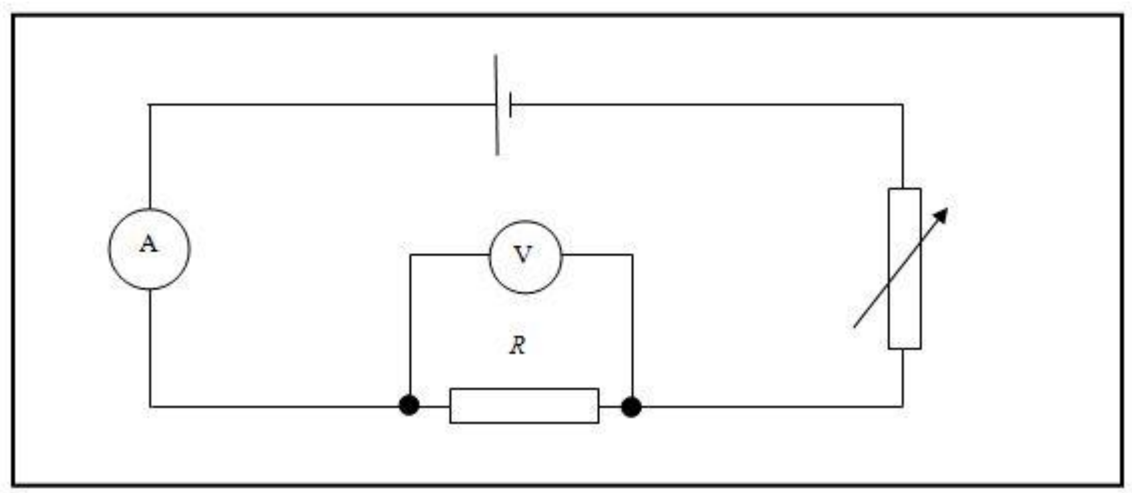


Figure 23 Circuit to harvest data to plot a voltage current graph.

The **variable resistor** is there to change the voltage and the current. A variable power supply (like a lab-pack) will also do the job, and a variable resistor is not needed. Remember that the voltmeter is connected in **parallel** across the component; the ammeter is connected in **series**.

The photograph (*Figure 24*) shows a typical experiment:

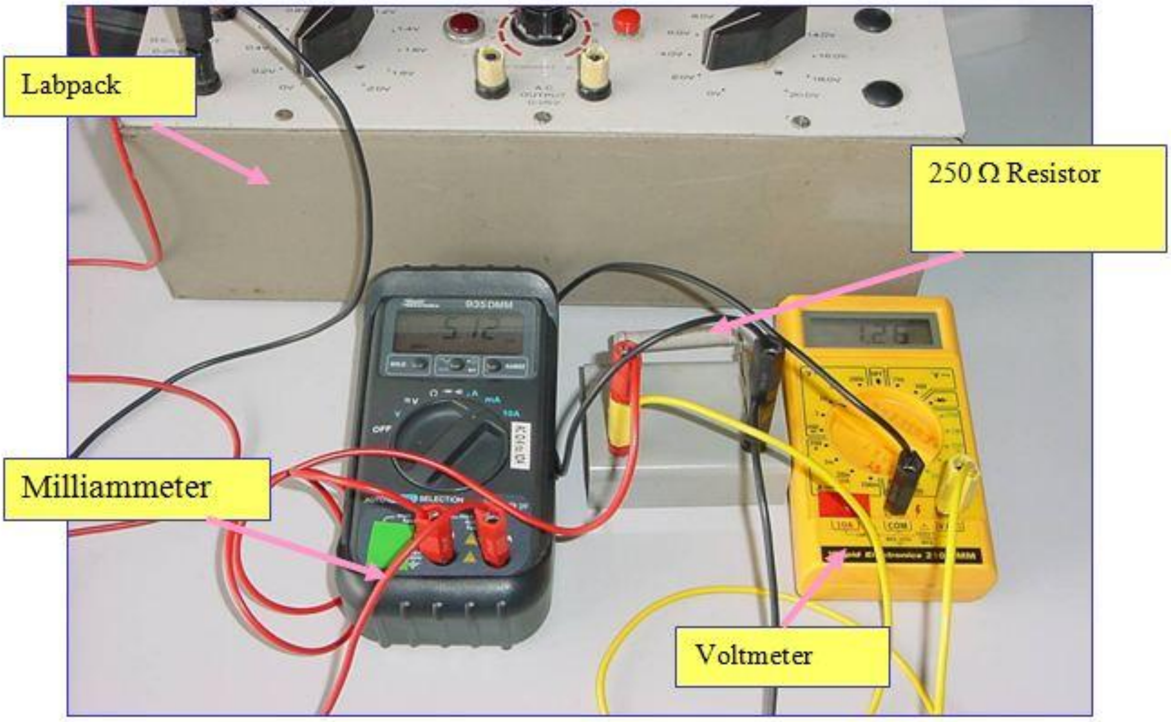


Figure 24 Measuring Voltage against Current.

The investigation of voltage-current characteristics lends itself well to **data-logging** techniques. The voltmeter and ammeter **sensors** are wired in exactly the same way as ordinary meters. They are then connected to the computer. If you see a question in the exam about data-logging, you should indicate clearly that the sensors are connected to the computer.

4.032 Voltage Current Characteristics

From this circuit we take readings of **voltage** and **current** plotting them as a graph called a **VI characteristic**.

We normally put the voltage on the y-axis and current on the x-axis. This allows us to determine the **resistance** from the **gradient**. If the voltage is plotted on the x-axis and the current on the y-axis, the gradient gives the conductance. This is a voltage current graph for an **ohmic** conductor (*Figure25*):

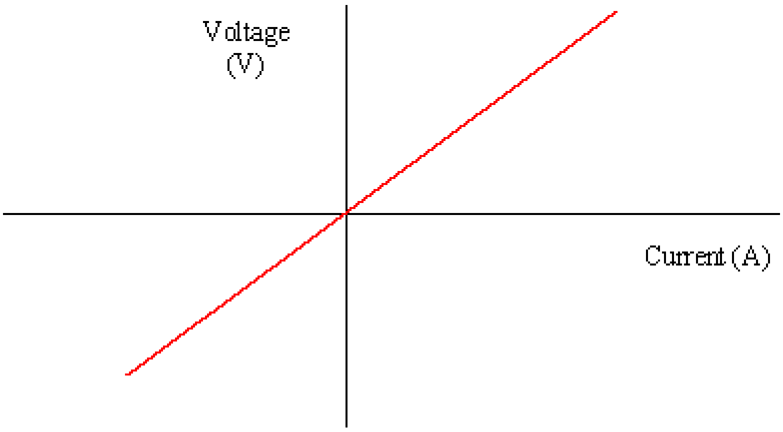


Figure 25 Voltage against current

The straight line shows a **constant ratio** between voltage and current, for both positive and negative values. We say that the voltage is **directly proportional** to the current. This means that the graph is a **straight line** of **positive gradient** going through the **origin**. So when the voltage is negative, the current is negative, i.e. flowing in the opposite direction. Ohm's Law is obeyed.

In the exam, you will, most likely, see that the **current is plotted against the voltage**. Therefore, the gradient is the **conductance**, and you have to find out the **reciprocal** to get the resistance. The higher the resistance, in this case, the less steep the gradient.

It is entirely possible that the graphs are plotted with the **voltage against the current**. Read the question carefully.

For a filament lamp we see:

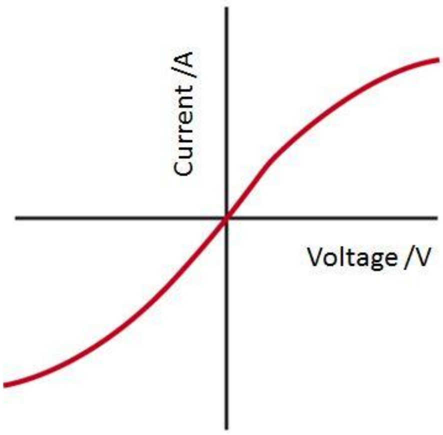


Figure 26 Current-Voltage characteristic for a filament lamp

The resistance **rises** as the filament gets **hotter**, which is shown by the **gradient getting steeper**.

4.033 Thermistor

A **thermistor** (a heat sensitive resistor) behaves in the opposite way. Its resistance goes **down** as it gets **hotter**. This is because the material releases more electrons to be able to conduct (*Figure 27*). Don't worry about why this happens; it's not on the syllabus.

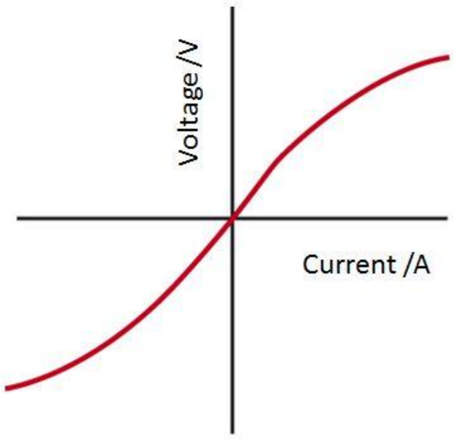


Figure 27 Voltage-current characteristic of a thermistor

Although it looks similar to the graph in *Figure 26*, notice how the gradient is decreasing, indicating a lower resistance. Note how the scales are the opposite way round in *Figure 27* compared to *Figure 26*.

Figure 27 shows that as the current goes up, the thermistor gets hotter. This can mean that:

- As it gets hotter, it allows more current to flow.
- Therefore, it gets hotter and so on.

This is called **thermal runaway** and is a feature of many semi-conductor components. At the extreme the component will glow red-hot, then split apart. Do NOT try it for yourself (unless you want an earful from your physics teacher, and, possibly, an interview with the vice-principal or the headmaster).

The thermistor is used wherever any electronic circuit detects temperature (*Figure 28*):

Figure 28 A thermistor protects an electronic circuit form overheating.

Here we see a thermistor protecting a power supply from too high a temperature.

You can investigate how temperature and resistance are related in a thermistor using equipment like this (*Figure 29*):

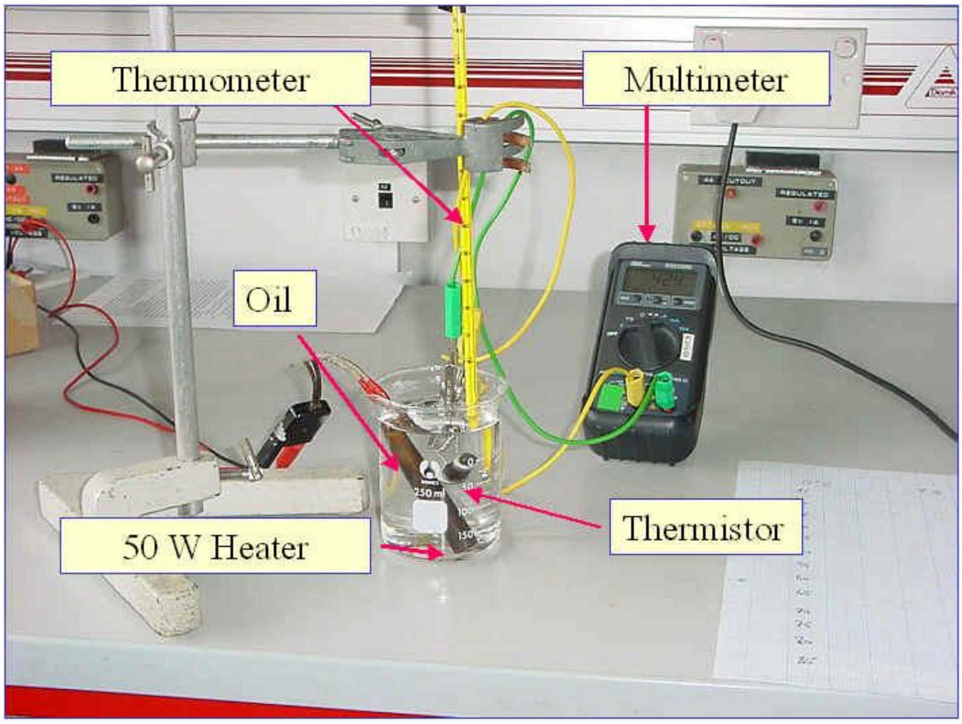


Figure 29 Experiment to investigate how the resistance of a thermistor changes with temperature.

4.034 Diodes

Diodes are semi-conductor devices that allow electric current to flow one way only.

- If the positive of the diode is connected to the positive terminal of the battery, the diode will conduct.
- The voltage across the diode is about 0.6 V.
- This is called **forward bias**.
- The negative terminal, sometimes called the cathode is shown by the silver band.

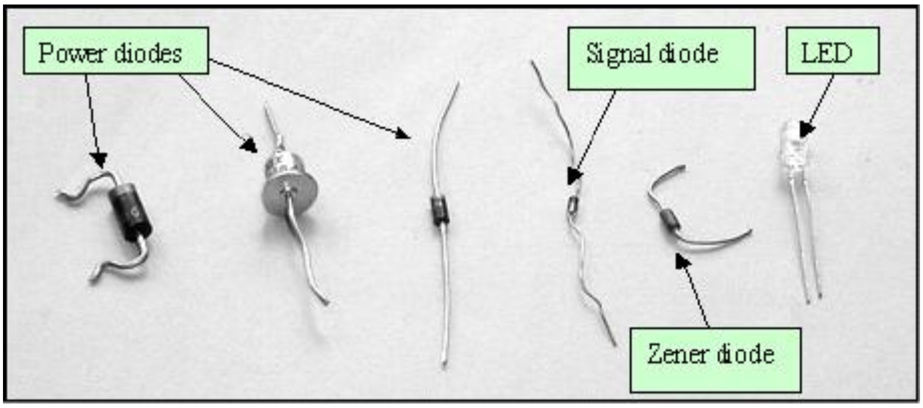


Figure 30 A selection of diodes

The circuit to measure the characteristic of a diode is like this, based on a **potentiometer** (*Figure 31*).

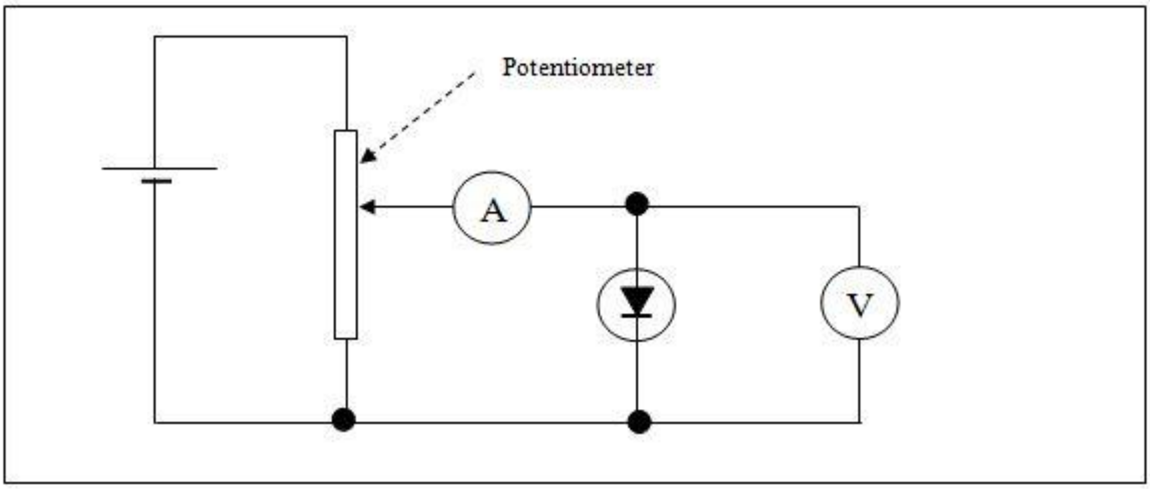


Figure 31 A circuit to measure the voltage current characteristic of a diode.

The **potentiometer** allows a range of values from 0 volts to the battery voltage. We will look at the potentiometer in more detail in a later tutorial.

The **diode characteristic graph** looks like this (*Figure 32*):

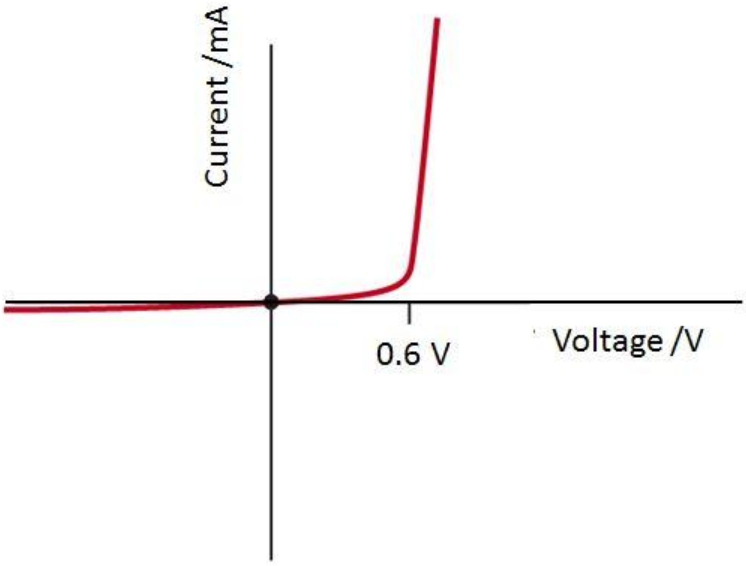


Figure 32 Diode characteristic graph

Most text-books show it like this, as do electronic engineers. However, you may see the graph plotted as **voltage against current**.

Here it is the other way round (*Figure33*):

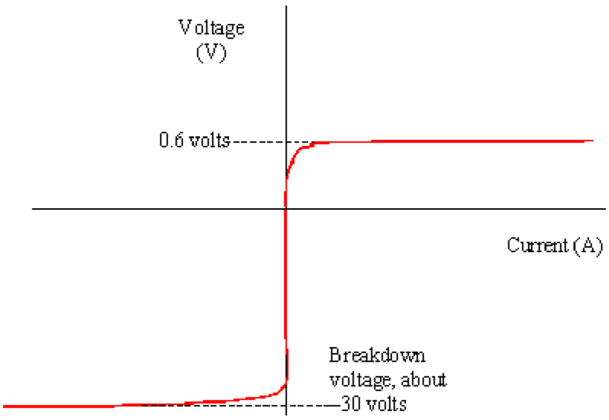


Figure 33 Alternative way of showing a diode characteristic

The diode starts to conduct at a voltage of about **+0.6 V**. We call this **forward bias**. Then the current rises rapidly for a small rise in voltage. If the current is reversed (**reverse bias**) almost no current flows until the **breakdown voltage** is reached. This usually results in destruction of the diode.

Tutorial 4.03 Questions

4.03.1

Can you explain why the shape of this graph suggests that a light bulb does not obey Ohm’s Law?

4.03.2

Why does a thermistor not obey Ohm's Law?

4.03.3

(Harder) Can you use the graph in *Figure 33* to explain why a diodes allows a current to flow one way only?

Tutorial 4.04 Resistivity	
All Syllabi	
Contents	
4.041 Resistivity	4.042 Measuring Resistivity
4.043 Conductivity	4.044 Super Conduction
4.045 Conduction in a vacuum	4.046 Ionic Conduction
4.047 Metallic Conduction	4.048 How fast do electrons move?

4.041 Resistivity

The resistance of a wire depends on three factors:

- the length; double the length, the resistance doubles.
- the area; double the area, the resistance halves.
- the material that the wire is made of.

Resistivity is a property of the material. It is defined as the **resistance of a wire of the material of unit area and unit length.**

The formula for resistivity is:

$$\text{Resistivity (ohm metres)} = \frac{\text{area (metres}^2) \times \text{resistance }(\Omega)}{\text{Length (m)}}$$

In physics code we write this as:

$$\rho = \frac{AR}{l}$$

.... *Equation 12*

We can rewrite this to give:

$$R = \frac{\rho l}{A}$$

.... Equation 13

There are three bear traps:

- The unit for resistivity is **ohm metre** (Ωm), NOT ohms per metre.
- Notice too that the physics code ρ (rho, a Greek letter 'r') is the same as that for density. Resistivity has NOTHING to do with density.
- The area is in **square metres**. Real wires have areas in square millimetres; 1 mm^2 = 1 x 10^{-6} m^2

4.042 Measuring Resistivity

This is a **required practical**. You will have a sample of a resistance wire (usually constantan or nichrome). You will need to measure the diameter of the wire with a **micrometer screw gauge**. This should be done in **three** separate places to reduce the uncertainty. You work out the area of the wire using:

$$A = \frac{\pi D^2}{4}$$

.... Equation 14

Then you will measure the resistance of lengths of the resistance wire, using a circuit like this (*Figure 34*).

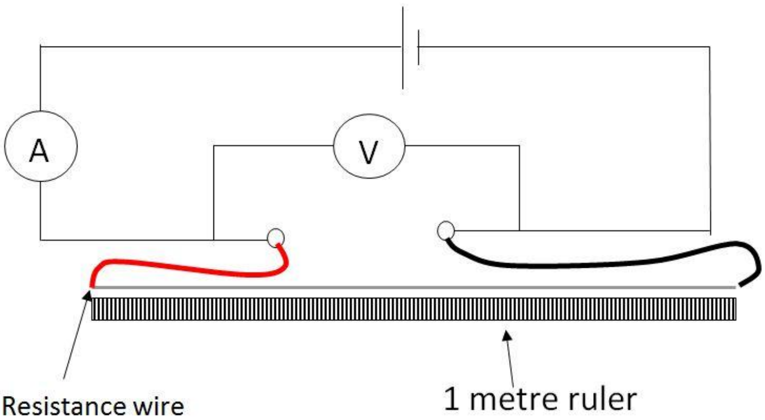


Figure 34 Measuring Resistivity of a wire.

You measure lengths of 0.10 m, 0.20 m, etc. You measure the voltage and current. You should do repeat readings and take averages. Then you need to process the data to get the resistance. (You know how to do this, don't you?) Then the data are plotted on a graph like this (*Figure 35*).

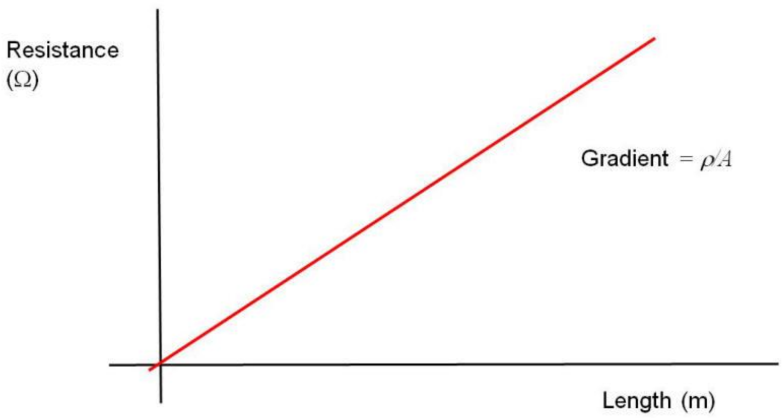


Figure 35 Graph showing variation of resistance with length.

The gradient is measured. To work out the resistivity, you need to multiply the gradient by the area of the wire. This can be compared with the data book value of the resistivity.

4.043 Conductivity

The **reciprocal** or **inverse** of resistivity is **conductivity**. It has the physics code σ, ("sigma", a Greek letter 's'), and units **Siemens per metre** (S m^{-1}).

$$\sigma = \frac{1}{\rho}$$

.... Equation 15

Conductivity is given by the relationship:

$$G = \frac{\sigma A}{l}$$

.... Equation 16

This simply an inversion of *Equation 10*.

4.044 Super-conduction

A **super-conductor** is a material that has **zero resistance**. A current flows when there is no potential difference. The piece of metal floating above the magnet shows that there must be a current flowing (*Figure 36*).

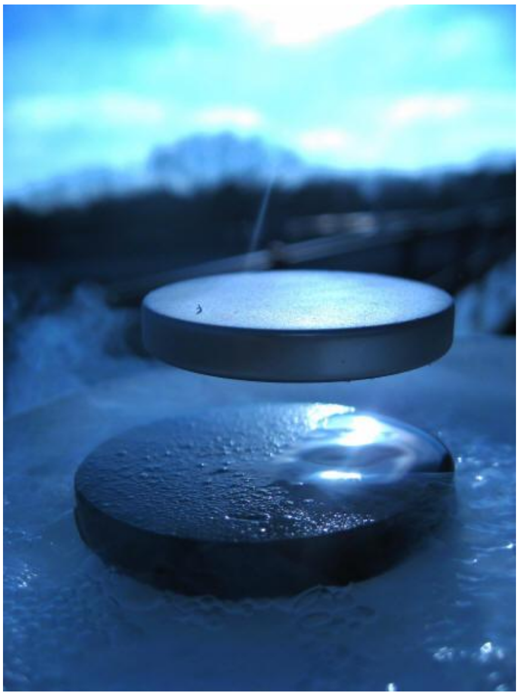


Figure 36 Superconductivity. Picture from Wikimedia Commons.

For all metals the resistivity (hence resistance) **decreases** as they get colder. For some metals like copper and silver, there is still a tiny bit of resistance left at very low temperatures. Very low temperatures have to be maintained, which is expensive. Room temperature superconductivity has not been seen.

Super-conductivity is seen in:

- Aluminium
- Tin.
- Some alloys.
- Some heavily doped semi-conductors.

All superconductors have a **critical temperature** above which the phenomenon stops. The graph below shows the idea (*Figure 37*).

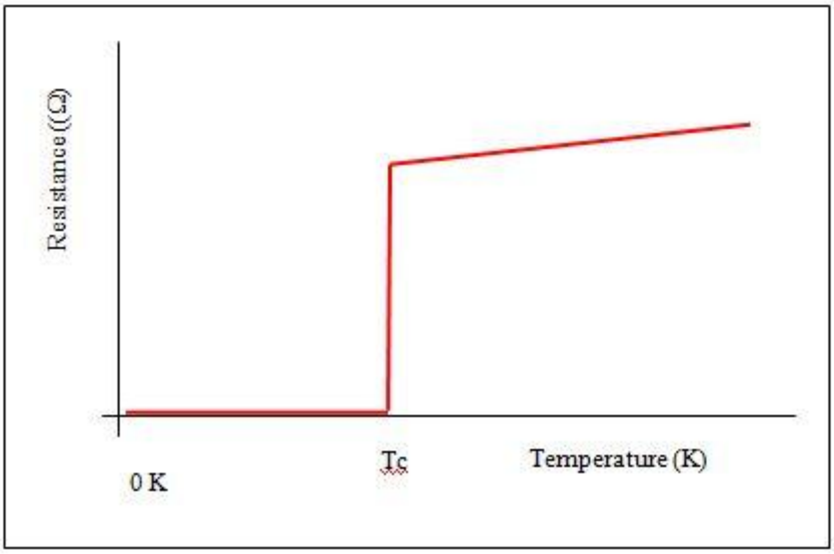


Figure 37 Critical Temperature in a superconductor.

Above the boiling point of liquid nitrogen, 77 K (-196 °C), superconductivity can be observed in a few materials. These are called **high temperature superconductors**.

Very large magnets such as those found in the large hadron collider have coils made of superconducting materials. It is believed that the superconductivity will last 100 000 years, as long as the coils don't go above their critical temperature.

The mechanism for super-conduction is complex and cannot be explained in terms of electrons colliding with ions. The Meisner Effect and flux trapping are not required on the syllabus.

Super-conduction is used in very powerful magnets used is MRI Scanners and machines used in high energy particle physics (e.g. cyclotrons, and accelerators).

4.045 Conduction in a vacuum

It's quite simple - there is no conduction in a **perfect** vacuum. There are no charge carriers to carry a current. It is a perfect **insulator**. High voltage switching is often carried out in a vacuum to prevent **arcing** between the poles of the switch. In real life, there are no such thing as a perfect vacuum (even outer space has about 1 atom per cubic metre). In the case of the switchgear, there may be a small spark as the poles of the switch open. This is because the vacuum will be far from perfect. It's better described as a low-pressure gas, and there are plenty of molecules.

4.046 Ionic Conduction (OCR syllabus, Eduqas, Edexcel and WJEC)

Electricity moves due to the movement of **charge carriers**. If we think about an **ionic** solution, the positive ions are attracted to the negative terminal (the **cathode**), while the negative ion are attracted to the positive terminal (the **anode**).

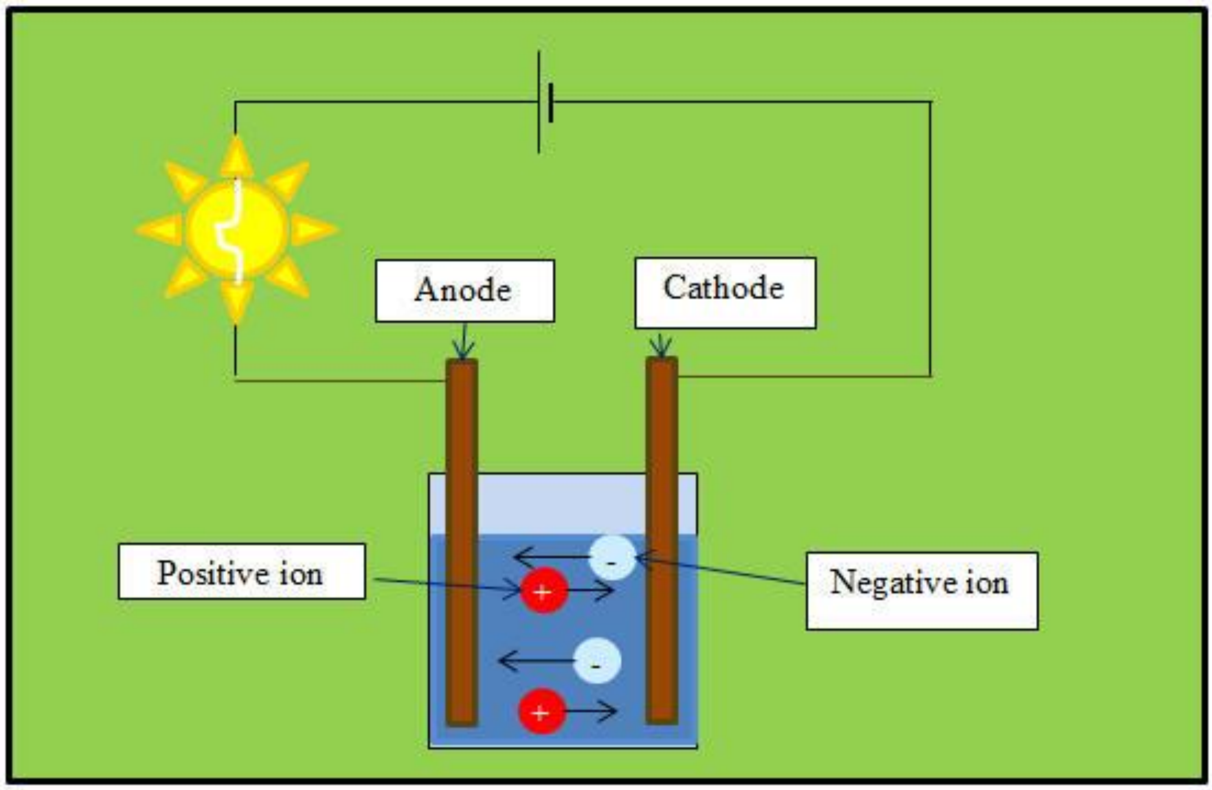


Figure 38 Charge carriers in a solution are attracted to electrodes of opposite polarity.

An **inactive** or **inert** electrode is made of an un-reactive metal, for example, platinum. It just transfers electrons.

An **active** electrode participates in any chemical reaction. For example, a copper anode (made of impure copper) in a copper sulphate solution takes up electrons from the solution. The copper metal loses its electrons to become copper ions.

$$Cu_{(s)} \rightarrow Cu^{2+}{}_{(aq)} + 2e^{-}$$

At the cathode, which attracts the positive copper ions, the copper ions gain electrons to become (pure) copper metal.

The voltage current graph for conduction in an ionic solution looks like this (*Figure 39*)

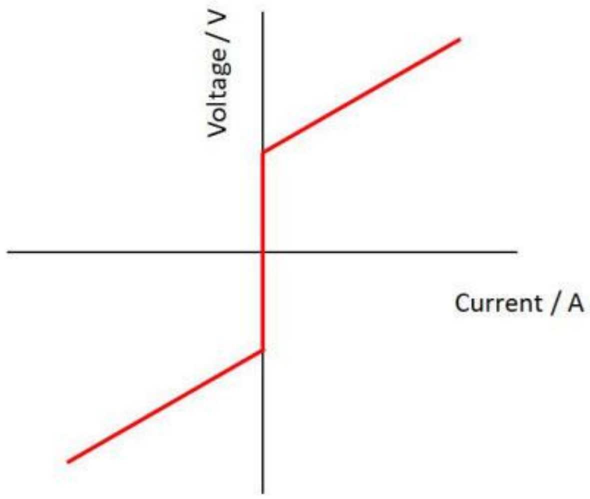


Figure 39 Voltage current graph for conduction in an ionic solution.

There is a range of negative to positive voltages between which no current flows. This is because a certain amount of work has to be done to separate the positive ion from the negative ion. Since voltage is work done per coulomb, the voltage represents the work that needs to be done.

A similar situation is found in gases at **low pressure**. At atmospheric pressure, gases are poor conductors (good insulators). However, if a high enough voltage is applied across a gas at low pressure, the gas becomes ionised and will conduct electricity. You will see the gas glow. If the voltage is high enough, all the electrons are removed from the nucleus. The gas is a **plasma**.

4.047 Metallic Conduction (OCR syllabus, Eduqas, Edexcel and WJEC)

In a **metallic conductor** (wire), the simplest model of conduction is to consider the metal as a **lattice of metal ions in a sea of free electrons**. The electrons move about **randomly** (*Figure 40*).

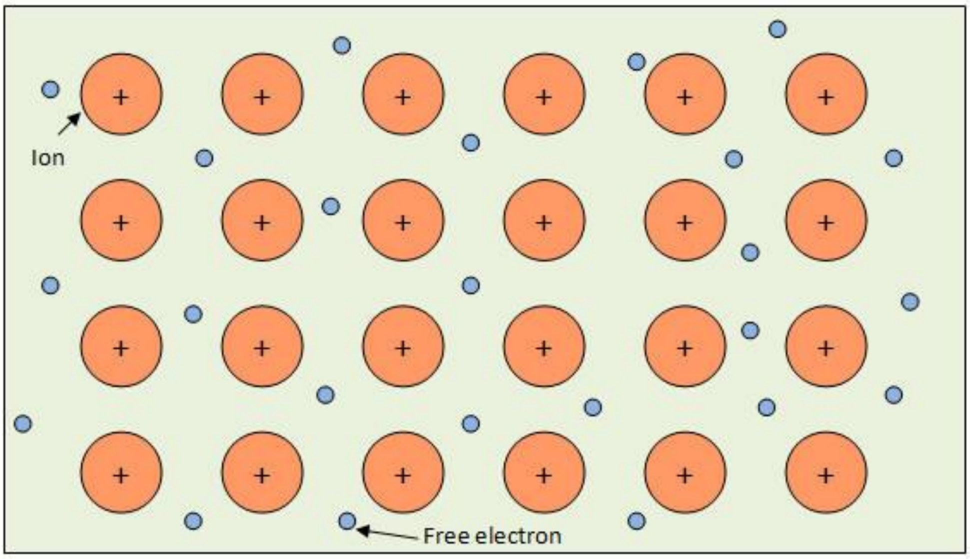


Figure 40 Conduction model for a metallic conductor

When a voltage is applied across the ends of the wire, the electrons continue to move randomly, but there is an overall drift to the positive end of the wire. So, you will (rightly) think that electrons go from negative to positive. The protons don't move. So, this idea is opposite to what you have been told. The explanation is that the earliest physicists got it wrong. They didn't know about electrons in the Eighteenth Century. So instead of rewriting all the rules of electricity, people talked about **conventional** current going from **positive to negative**. All currents are regarded as conventional.

The voltage-current graph (*Figure 41*) for a metallic conductor shows direct proportionality (as long as the temperature and other physical conditions remain the same).

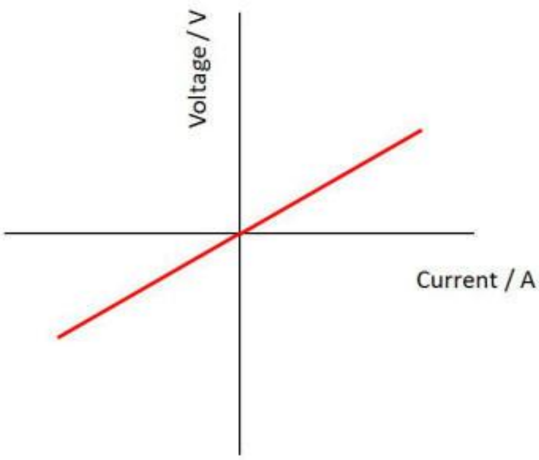


Figure 41 Voltage current graph for a metallic conductor.

Resistivity is the **resistance of a wire of 1 metre length and of 1 m^2 cross sectional area**. According to this model, resistance arises due to the **vibration of the metal ions**, and the probability of the ions colliding with the free electrons. Resistance is a representation of the **probability of collisions**. Such collisions involve a change of energy from **kinetic** energy to **internal** energy. The increase in internal energy makes the **temperature** in the ions rise, hence increases the probability of a collision. Therefore, the resistance increases as the wire gets hotter.

4.048 How fast do electrons move? (OCR, Eduqas, Edexcel and WJEC)

When a voltage is applied across the ends of the wire, the electrons continue to move **randomly**, but there is an **overall drift** to the positive end of the wire. So, you will (rightly) think that electrons go from negative to positive. The protons don't move. You will remember that **conventional current** flows from positive to negative.

We can write an equation for the conduction of a current in a wire. The current depends on:

- The speed of the charge carriers (v (m s^{-1})).
- The area of the wire (A (m^2)).
- The charge on the electrons (e = 1.6 × 10^{-19} C).
- The number of charge carriers per unit volume (N (m^{-3})).

The symbols in *italics* are the physics codes for the various quantities, and the units are given as well.

The formula is:

$$I = NAve$$ .... *Equation 17*

The **number of charge carriers per unit volume** is probably the hardest quantity to get your head round. It means the **number of electrons in a cubic metre** of the material. For example, copper has 8 × 10^{28}electrons in each cubic metre of material. You can look up the number of electrons per cubic metre in a data book.

Derivation

Consider a piece of metal that is l m long and has an area of A m^2. It is made of a metal that has N free electrons m^{-3}. Let's suppose that a current of I A is flowing. Each electron has a charge of e C. See *Figure 42*.

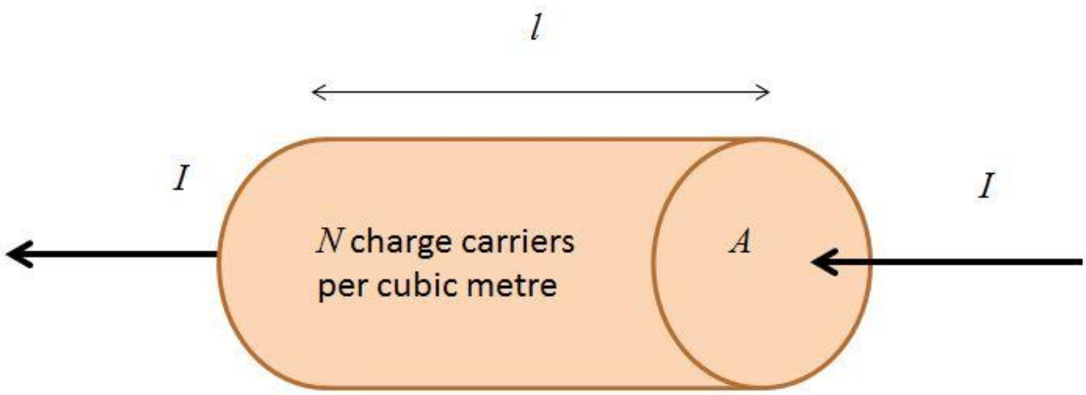


Figure 42 Current I flowing in a conductor of area A, of length l and charge carriers per unit volume, n.

We know that current is the flow of charge every second.

$$I = \frac{Q}{t}$$

.... *Equation 18*

The total charge is the number of electrons multiplied by the volume of the metal and the charge on each electron:

$$Q = NAle$$

.... *Equation 19*

So, we can substitute for *Q*:

$$I = \frac{NAle}{t}$$

.... *Equation 20*

The term l/t is a distance over time which is **speed**, v m s^{-1}. So, we write:

$$I = NAve$$ ....*Equation 21*

Worked Example
A wire is carrying a current of 200 A. Its cross-sectional area is 7.85×10^{-5} m^2. If copper has 8.0×10^{28} electrons per cubic metre, what is the speed of the electron drift? Give your answer to an appropriate number of significant figures.
Answer Rearrange the *Equation 18*: $$v = \frac{I}{nAe}$$ v = 200 A ÷ (8.0×10^{28} m^{-3} × 7.85×10^{-5} m^2 × 1.6×10^{-19} C) = **2.0×10^{-4} m s^{-1}** The answer is given to 2 significant figures as the data are given to two significant figures.

The result from this calculation shows that the speed of electron drift is **slow**, 1 mm every 5 seconds. The propagation of the message "that the current is flowing" is very fast, not far off the speed of light. But the **drift speed** of electrons is not at all fast.

We can show this by putting a small crystal of potassium permanganate (a dark purple ionic compound) onto a filter paper which has been soaked in sodium chloride solution.

When a current flows, the purple permanganate ions move towards the positive terminal, but it takes some time.

In questions on this, you may be given a **diameter** of a wire.

You must convert millimetres to metres when working out the area and you must be sure whether you are given the diameter or radius.

$$A = \frac{\pi d^2}{4}$$

The tungsten filament has a higher resistance than the copper wire. For the same current to pass through, the electrons have to go **faster**, because:

- The number of charge carriers per cubic metre is smaller.
- The area is smaller.

This is opposite to the generally perceived idea that electrons are slowed down by high resistances. There is a greater chance of a collision, so the wire will hotter.

Models of **semiconduction** are to be found in a later Tutorial, **4.06**.

Tutorial 4.04 Questions

4.04.1

Constantan has a resistivity of $47 \times 10^{-8}\ \Omega$ m. How much of this wire is needed to make a 10 Ω resistor, if the diameter is 0.50 mm? Give your answer to an appropriate number of significant figures.

4.04.2

Explain what happens when a super-conducting metal reaches its critical temperature.

4.04.3

A copper wire of diameter 1.4 mm connects to the tungsten filament of a light bulb of which the diameter is 0.020 mm. The current in both materials is 0.52 A. Find the speed of an electron in each of the two materials.

Copper has 8×10^{28} electrons per cubic metre. Tungsten has $3.4 \times 10^{28}\ m^{-3}$.

Tutorial 4.05 Energy and Power in Circuits

All Syllabi

Contents

4.051 Conventional Current

Conventional current goes from **positive** to **negative**. **Electrons** carry energy around the circuit; they go from **negative** to **positive**. In the early days, physicists didn't know about the electron, which is why they got it all wrong. Correction would require a complex re-write of the Laws of Physics, a task which no-one is likely to want to tackle. So, all conventional currents are from positive to negative. All currents are treated as **conventional**.

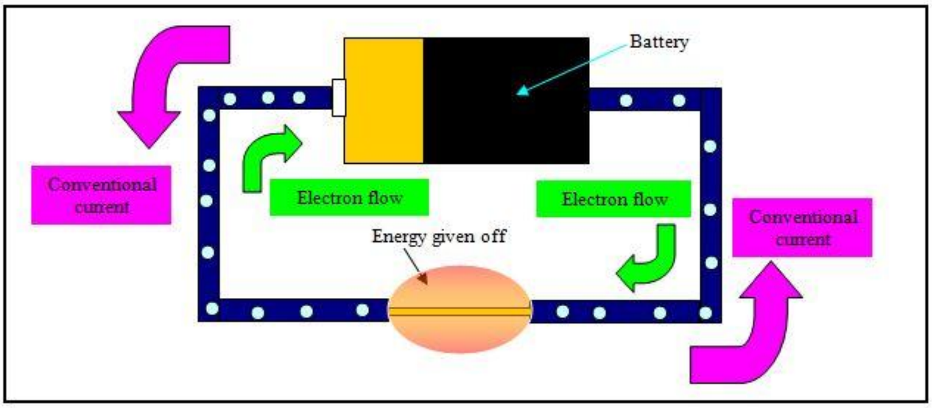


Figure 43 Illustrating the concept of conventional current

We can measure the energy in a circuit by measuring the voltage and the current (*Figure 44*).

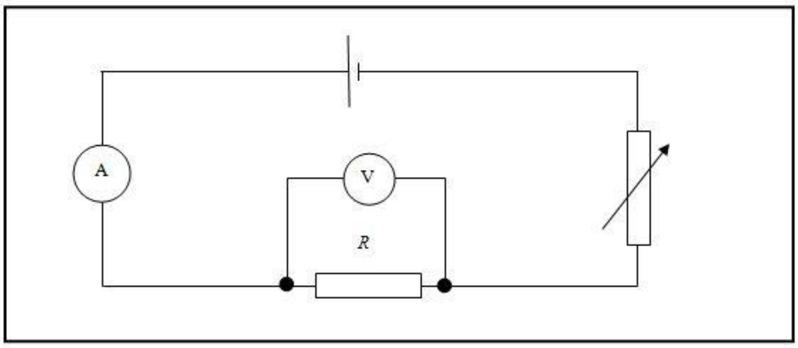


Figure 44 Measuring voltage and current

The voltage current graph looks like this (*Figure 45)*

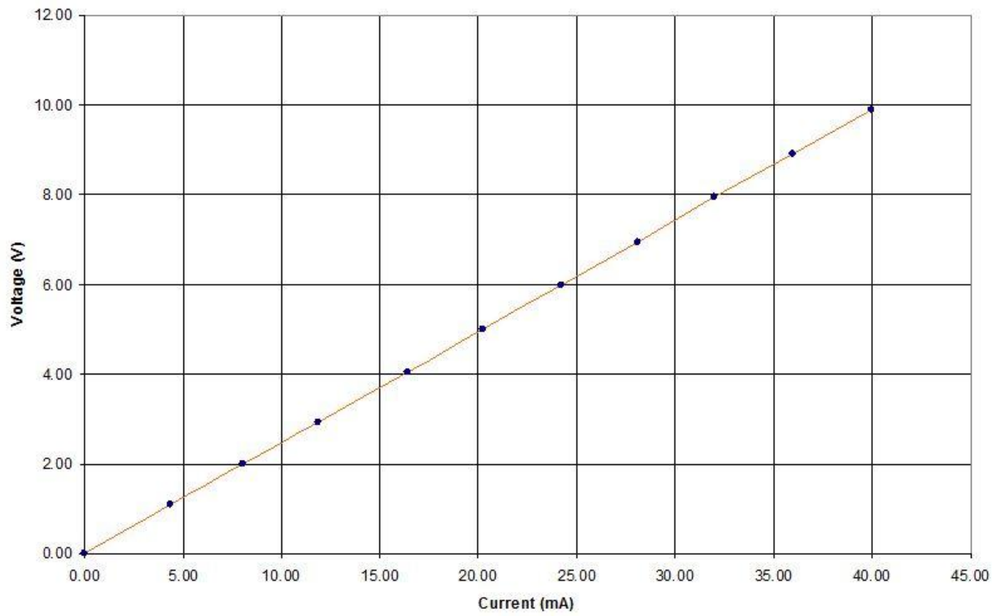


Figure 45 Voltage current graph for a resistor

4,042 Power in a Current

Suppose a current I amps flows for t seconds in a component. The charge that flowed led to E joules being dissipated in the component.

We know that:

$$Q = It$$ .... *Equation 22*

$$E = QV$$ .... *Equation 23*

So, if we substitute Q out of *Equation 20*, we get:

$$E = ItV$$ .... *Equation 24*

Now

$$\text{Power} = \frac{\text{energy}}{\text{time}}$$

So, we can write:

$$P = \frac{ItV}{t}$$ .... *Equation 25*

It doesn't take a genius to see that the term t cancels out to leave us with:

$$P = IV$$ .... *Equation 26*

Power is measured in **watts** (W). 1 watt = 1 joule per second

4.053 The Heating Effect of a Current

We know that:

$$V = IR$$ .... *Equation 27*

$$P = VI$$ .... *Equation 28*

So, we can write:

$$P = I \times IR$$ .... *Equation 29*

So it doesn't take a genius to see that by substituting *Equation 25* into *24*, we get:

$$P = I^2R$$ .... *Equation 30*

We can use a similar method to see how voltage, resistance and power are related

We know that:

$$I = \frac{V}{R}$$ .... *Equation 31*

$$P = VI$$ .... *Equation 32*

So, we can write:

$$P = V \times \frac{V}{R}$$ ....*Equation 33*

So, it doesn't take a genius to see that by substituting *Equation 30* into *29*, we get:

$$P = \frac{V^2}{R}$$ .... *Equation 34*

The graph of **power** against **current** looks like this (*Figure 46*):

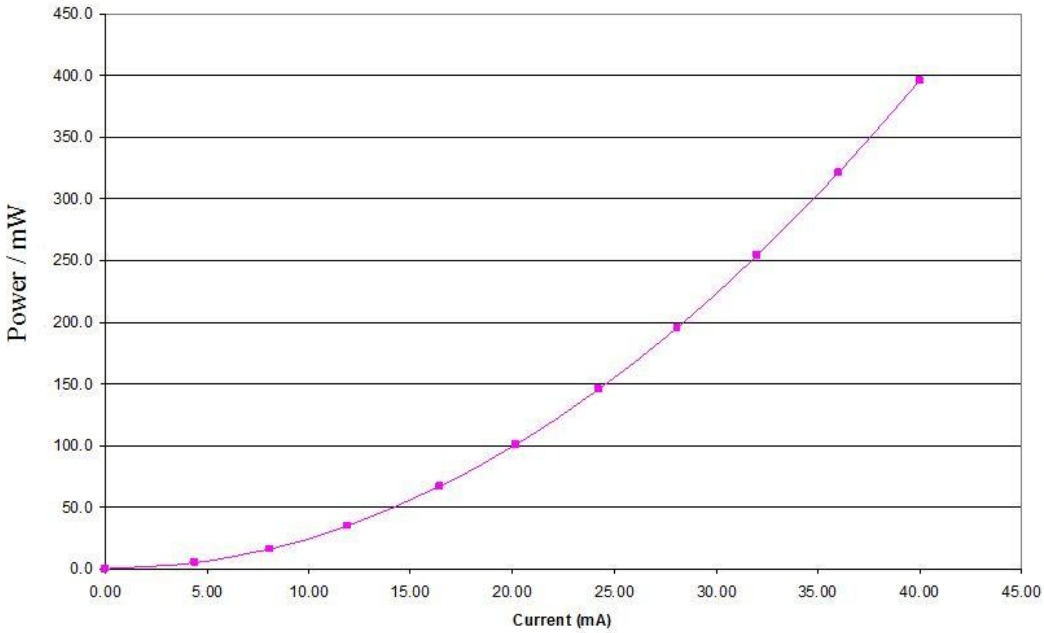


Figure 46 Graph of power against current

The graph shows that if we double the current, we get four times the power, consistent with the idea that $P \propto I^2$. If we were to plot P against I^2 we would get a straight-line graph.

The picture (*Figure 47*) shows the heating effect of a current on a resistor!

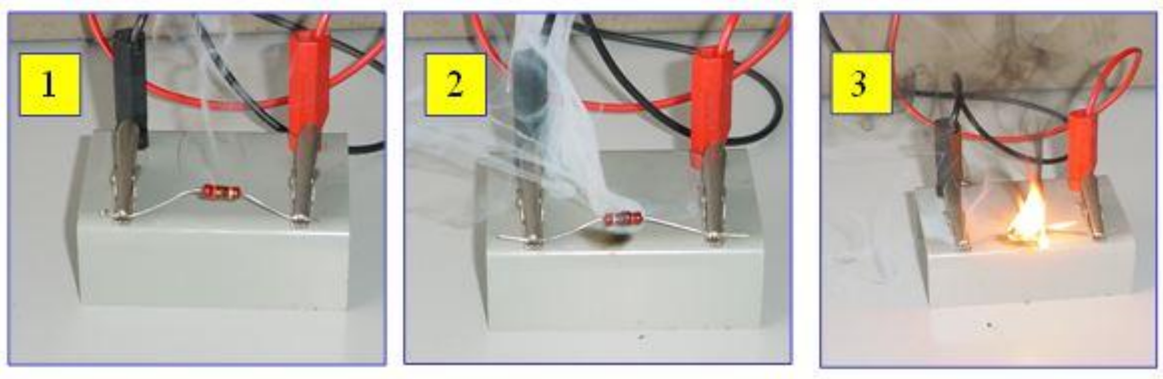


Figure 47 Effect of too much current in a resistor

This was a 33 Ω resistor connected to a 20 V supply. The current would be 20 V ÷ 33 Ω = 0.61 A

The power would be 0.61 A × 20 V = 12 watts. Plenty enough to fry a 1 watt resistor.

It is important that we ensure that any current limiting resistors can dissipate the power through them. The above situation could be **highly dangerous**.

Energy used or **work done** can be worked out by multiplying the **power** by **time**.

Therefore:

$$W = I^2Rt$$ .... *Equation 35*

4.054 Buying Electrical Energy (OCR)

In the early days of nuclear power, some people said that electricity would be in such plentiful supply that it would not be necessary to meter it. Back to reality. It costs a lot of money to generate and supply energy, and it need to be paid for.

Electricity is sold by the **kilowatt-hour** (kW h) which is defined as:

The amount of energy used by an appliance taking a power of 1 kW running continuously for 1 hour.

Not kilowatt per hour (kW h^{-1})

The kilowatt-hour is often referred to as a **unit** on an electricity bill. The average cost per unit as this is being written is about 26 p.

On all appliances there is a plate that shows the power consumption. This one is on a tumble dryer (*Figure 48*).

Figure 48 Power rating for an appliance.

This tumble dryer takes a power of 2300 W.

In the electricity bill, there is also a **standing charge**. This covers the cost of the infrastructure to supply your house with electricity. It is the same regardless of how much or how little electricity you use. Depending how you pay for your electricity, the standing charge is about 51 p a day. This works out at about £51 a quarter (90 days).

Large users of electrical energy pay in units of **megawatt-hours** (MW h) and the cost is about £90 per megawatt hour (9 p per kilowatt-hour). (This is, surprisingly, about the same as when I last rewrote these notes about seven years ago.)

4.055 Chemical Effects of an Electrical Current (Extension)

All batteries use a chemical reaction to produce a potential difference. If the terminals are connected to a circuit, a current will flow. For example, a car battery has an anode made of lead and a cathode of lead oxide in an **electrolyte** of **sulphuric acid**. There are two reactions that occur to make a current flow:

$$PbO_2 + 4H^+ + SO_4{}^{2-} + 2e^- \rightarrow PbSO_4 + 2H_20$$

and

$$Pb^{2+} + SO_4{}^{2-} \rightarrow PbSO_4 + 2e^-$$

The first reaction **absorbs** electrons to make the anode **positive**. The second reaction **produces** electrons to make the cathode **negative**. The electrons do not move through the electrolyte of the battery, but through the external circuit.

When the battery is charged up using an electric current, the reactions are reversed, to produce lead oxide. Hydrogen gas is also produced, which is why it's not a good idea to have naked flames around car batteries.

The chemical process of **electrolysis** is another example of electric currents being used to drive chemical reactions. Many electrolysis reactions are strongly **endothermic**, so need the external electrical energy to allow them to proceed.

Tutorial 4.05 Questions

4.05.1

A 12 volt heater takes a current of 3.6 A. It is left to heat up an aluminium block for a period of 45 minutes. How much heat energy is transferred to the aluminium block?

4.05.2

What current is consumed by a 60 W light bulb operating on the 230 V mains?

4.05.3

A resistor of value 50 ohms is rated at 1 watt. This means that if it has to give out more power than 1 watt, it will start to get hot. What is the maximum current that it can handle?

4.05.4

The same resistor as in 4.05.3 is then connected to a 20 volt supply. What power will it dissipate now? What do you think will happen to the resistor?

4.05.5

Convert 1 kW h to the equivalent energy in joules (J)

4.05.6

It takes 75 minutes to dry a load of washing. How much does this cost? Cost per unit is 26 p (at the time of writing).

4.05.7

At the start of a quarterly billing period (3 months or 90 days) the electricity meter reads 025628 kW h.

At the end it reads 026344 kW h.

If electricity costs 26 p per unit, calculate the electricity bill including the standing charge (£53 a quarter for this supplier).

Tutorial 4.06 Series and Parallel Circuits	
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4.061 Series Circuits

In a **series** circuit, the electrons in the current have to pass through all the components, which are arranged in a line. Consider a typical series circuit in which there are three resistors of value R_1, R_2, and R_3. The values may be the same, or different (*Figure 49*).

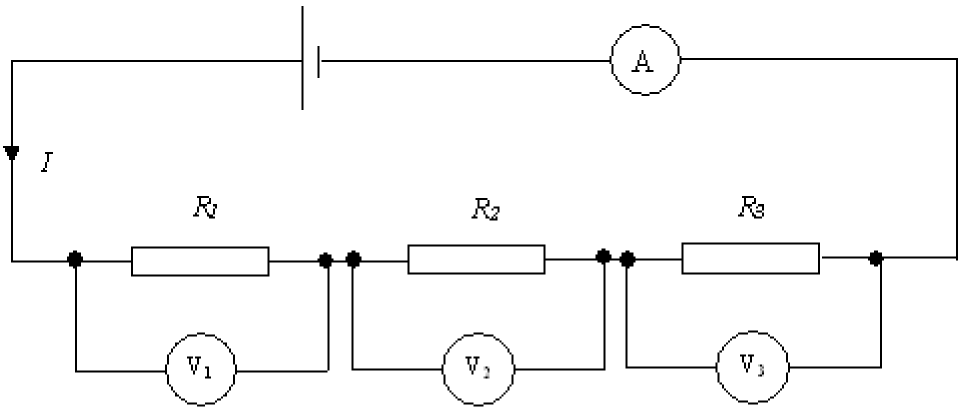


Figure 49 A series circuit

There are two key points about a series circuit:

- The **current** throughout the circuit is the same
- The **voltages** add up to the battery voltage.

Therefore:

$$V_T = V_1 + V_2 + V_3$$ .... *Equation 36*

From Ohm's Law we know:

- $V_T = IR_T$.
- $V_1 = IR_1$.
- $V_2 = IR_2$.
- $V_3 = IR_3$.

So, we can write:

$$IR_T = IR_1 + IR_2 + IR_3 \ldots\ldots\ldots \quad \textit{Equation 37}$$

Therefore, since the current, I, is the same throughout the circuit, we can write:

$$R_T = R_1 + R_2 + R_3 \ldots\ldots\ldots \quad \textit{Equation 38}$$

This is true for any number of resistors in series.

4.062 Parallel Circuits

Parallel circuits have their components in parallel **branches** so that an individual electron can go through one of the branches, but not the others. The current splits into the number of branches there are. Look at this circuit (*Figure 50*).

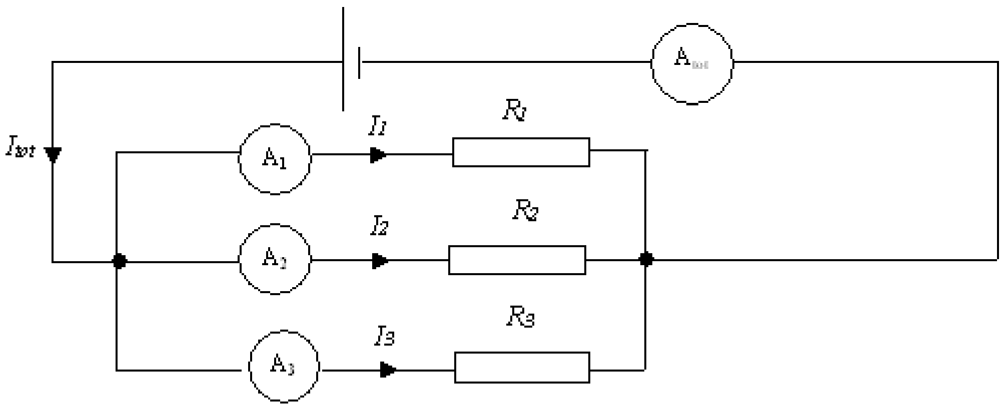


Figure 50 A parallel circuit

In this case, the current will split into three. For a parallel circuit we know two things:

- The voltage across each branch is the same
- The currents in each branch add up to the total current.

From this we can write:

$$I_T = I_1 + I_2 + I_3$$ .... *Equation 39*

From Ohm's Law, $I = V/R$, we can write:

$$I_T = \frac{V}{R_T}$$

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$

We can then write:

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

.... *Equation 40*

Since the voltage across each resistor is the same, we can write:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

.... *Equation 41*

This is true for any number of parallel resistors.

We can also write this as:

$$R_T^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$$

............*Equation 42*

We are adding up the **reciprocals** of the resistance.

Once you have done the addition of the reciprocals, remember to take the **reciprocal** of the answer. This will give you the resistance.

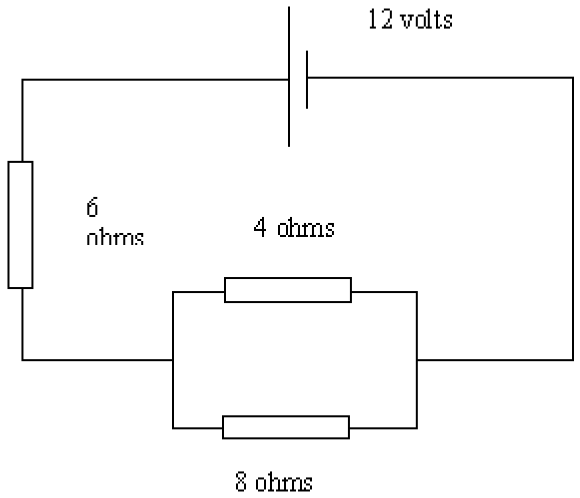
We can combine resistors in **both series and parallel**. Tackle the problem step by step.

- Work out the total resistance of the parallel combination.
- Work out the total resistance of the circuit by adding your answer in the previous step to the values of the series resistors.

Here is a worked example:

Worked Example

Look at this circuit:



What is the single resistor equivalent?

What is the total current?

What is the voltage across the 6 ohm resistor?

What is the current in each resistor?

Answer

What is the single resistor equivalent of the parallel resistor combination?

We will do the parallel combination first:

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4\ \Omega} + \frac{1}{8\ \Omega} = \frac{3}{8\ \Omega}$$

R_t = 8 Ω/3 = **2.67 Ω**

Now we can work out the overall resistance:

The overall resistance = 6 ohms + 2.67 ohms = **8.67 Ω**

What is the total current?

$I = V/R$ = 12 volts ÷ 8.67 Ω = **1.38 A**

What is the voltage across the 6 ohm resistor?

$V = IR$ = 1.38 amps × 6 Ω = **8.30 V**

What is the current in each resistor?

We need to know the voltage across the parallel resistors:

Voltage = 12 volts - 8.30 volts = 3.70 volts

Now we can work out the current in each branch because the voltage across each resistor is the same.

For the 4 ohm resistor:

$I = V/R$ = 3.70 volts ÷ 4 ohms = **0.93 A**

For the 8 ohm resistor:

$I = V/R$ = 3.70 volts ÷ 8 ohms = **0.46 A**

These two currents add up to **1.39** amps, but there are **rounding errors**. Watch out for this, but don't worry too much about them.

Take care with such problems:

- Make sure that the voltages across each part of the circuit add up to the battery voltage.
- Make sure the currents in the parallel part of the circuit add up to the battery current.
- If they don’t, go back and check what you’ve done wrong!

We often refer to the total resistance of a circuit as a single **resistor equivalent**. Resistors are available in certain values. One example is the E24 series, in which there are 24 values available in each decade (1, 10, 100, 1000, 10 000, etc.).

4.063 Kirchhoff's Laws

These two simple laws were drawn up in the Nineteenth Century by Gustav Robert Kirchhoff. They explain all observations we see in electric circuits. We can explain everything we have looked at in series and parallel circuits in terms of the two laws. They can also be used to explain more difficult circuits which cannot be explained in terms of simple series and parallel circuits.

Kirchhoff I

This states that:

the total current flowing into a point is equal to the current flowing out of that point.

In other words, the charge does not leak out or accumulate at that point. Charge that flows away must be replaced (*Figure 51*). It is **conserved**.

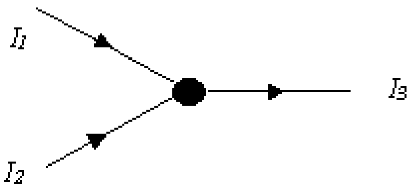


Figure 51 Kirchhoff I

From this diagram we can easily see that

$$I_3 = I_1 + I_2 \text{ ………….. } \textit{Equation 43}$$

Mathematically we can write this as:

$$I_1 + I_2 + -I_3 = 0 \text{ …………… } \textit{Equation 44}$$

Notice that I_3 has a minus sign. This means that the current going out is regarded as negative while current coming in is positive. At no point is there any reference to charge pooling at the junction, for the simple reason that it does not.

In some text books you will see written $\Sigma I = 0$. The strange looking symbol Σ is Sigma, a Greek capital letter S, which means "sum of". So, the sum of currents is zero, as we have seen above.

Kirchhoff II

Kirchhoff's Second Law is not quite so easy to grasp. It states:

The potential differences around a circuit add up to zero.

Provided the charge returns to the same place as it started, the gains and losses are equal, **no matter what route is taken by the charge**. The battery in this circuit has an emf (electromotive force or open terminal voltage) of $\mathcal{E}$. The curly E is the battery voltage. We will look at emf later.

Let us do a journey around the circuit from A to B to C, and back to A (*Figure 52*).

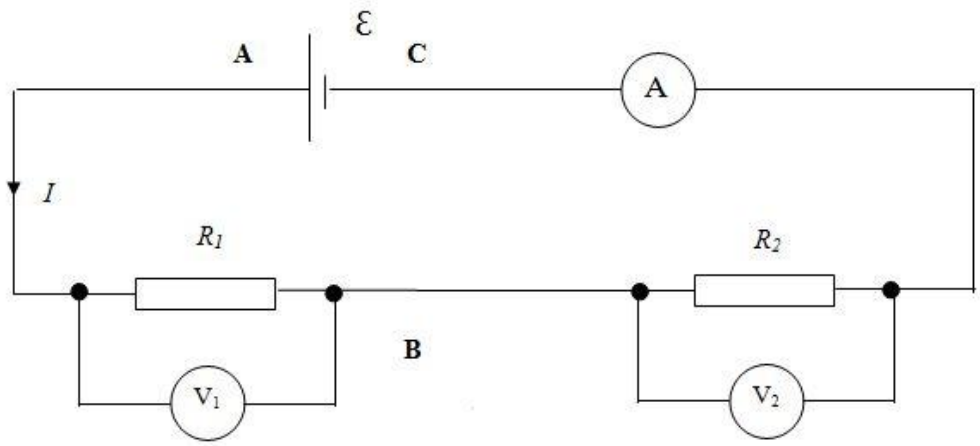


Figure 52 A circuit to show Kirchhoff II

- From A to B the p.d. change is IR_1 volts
- From B to C the p.d. change is IR_2 volts
- From C to A the p.d. change is $\mathcal{E}$ volts.

If we add up all the voltages, we can write:

$$IR_1 + IR_2 = \mathcal{E} \quad \text{........... Equation 45}$$

or

$$IR_1 + IR_2 + -\mathcal{E} = 0 \quad \text{............ Equation 46}$$

or

$$\Sigma(\text{p.d.}) = 0$$

This is another way of saying that the **voltages add up to the battery voltage**.

Although the statement above may seem obvious enough to some, Kirchhoff II does throw a lot of students, so let's have a further look (*Figure 53*).

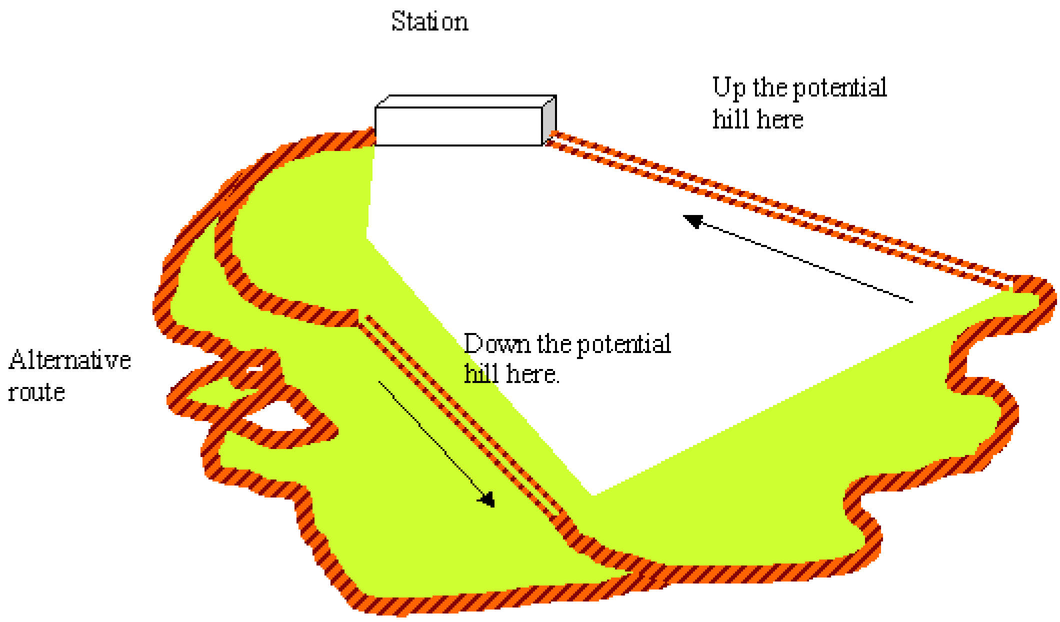


Figure 53 Showing the idea of the potential hill

The picture shows a simple roller-coaster railway. The cars leave the station. They can go downhill the straight way or the wiggly way. It doesn't matter. They then have to go back up the potential hill to reach the station they started from.

Some rollercoasters have several hills the cars go up. As they go up their potential increases. It's like a circuit with several batteries. You go up the potential hill as you go pass each battery. When you go up the potential hill, the sign is negative.

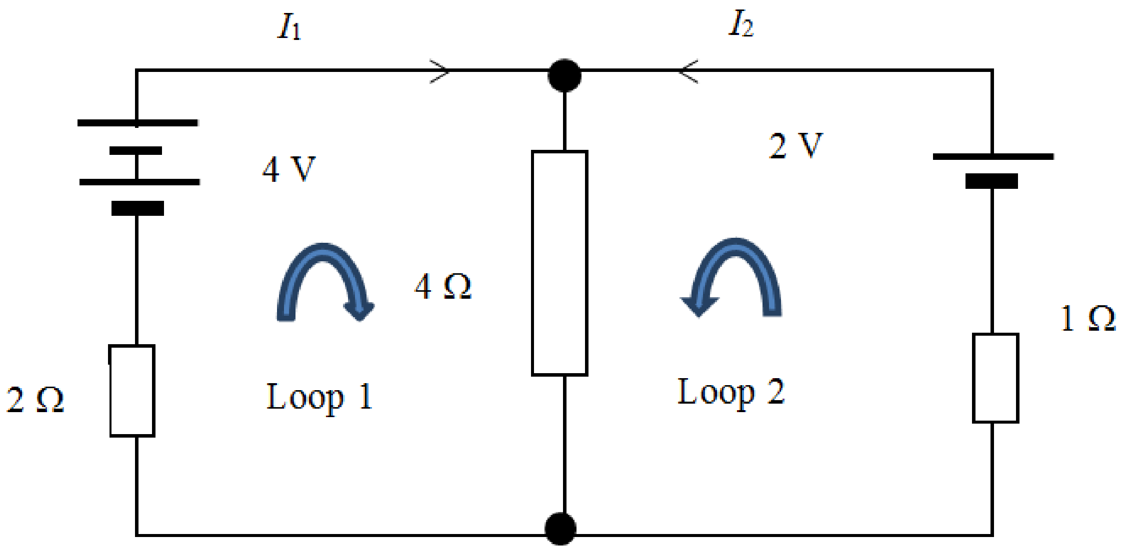
4.064 Using Kirchhoff's Laws (Extension for A-level and IB students)

Kirchhoff laws allow us to analyse some scary looking circuits. Adopt this problem-solving strategy:

- Break the circuit into loops.
- Work out the EMF for each loop.
- Work out the current in each loop.
- Sum the currents using Kirchhoff I;
- Sum the voltages using Kirchhoff II.

Worked example

Use Kirchhoff's Laws to work out the currents flowing in each branch of this network.



Answer

Identify loops 1 and 2. Go with the **conventional** current, so that Loop 1 is clockwise and loop 2 is anticlockwise.

EMF for Loop1 is 4 V, EMF for Loop 2 is 2 V (Kirchhoff II)

Sum the IR for loop 1:

Sum $IR = (I_1 + I_2) \times 4\ \Omega + I_1 \times 2\ \Omega = 4$ V

Sum the IR for loop 2:

Sum $IR = (I_1 + I_2) \times 4\ \Omega + I_2 = 2$ V (Note that we have I_2 passing through a $1\ \Omega$ resistor.)

Therefore:

$$4I_1 + 2I_1 + 4I_2 = 4 \text{ V}$$

so

$$6I_1 + 4I_2 = 4 \text{ V}$$

Also:

$$4I_1 + 4I_2 + I_2 = 2\text{ V}$$

so

$$4I_1 + 5I_2 = 2\text{ V}$$

Therefore, we have two simultaneous equations:

$$6I_1 + 4I_2 = 4\text{ V}$$

$$4I_1 + 5I_2 = 2\text{ V}$$

Multiply the first by 2 and the second by 3

$$12I_1 + 8I_2 = 8$$

$$12I_1 + 15I_2 = 6$$

Subtract the two equations from each other to get rid of $12I_1$

$$-7I_2 = 2$$

$$I_2 = -0.286\text{ A}$$

Now substitute into the second equation to get I_1:

$$4I_1 + 5 \times -0.286 = 2$$

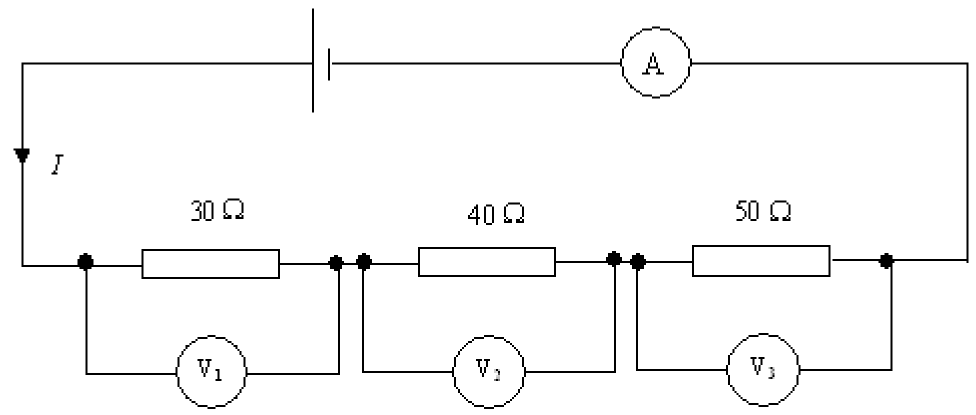
$$I_1 = \frac{3.43}{4} = 0.858\text{ A}$$

Total Current = 0.858 A – 0.286 A = **0.572 A** = 0.57 A (2 s.f.)

Tutorial 4.06 Questions

4.06.1

This question refers to the circuit below in which the current is 100 mA:



(a) What is 100 mA in amps?

(b) What is the current in each resistor?

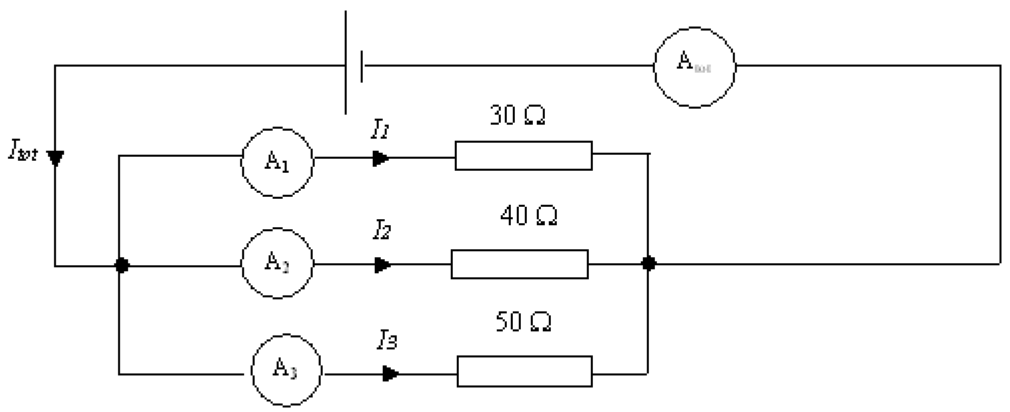
(c) What is the voltage across each resistor?

(d) What is the total resistance?

(e) What is the battery voltage?

4.06.2

This question refers to the circuit below.



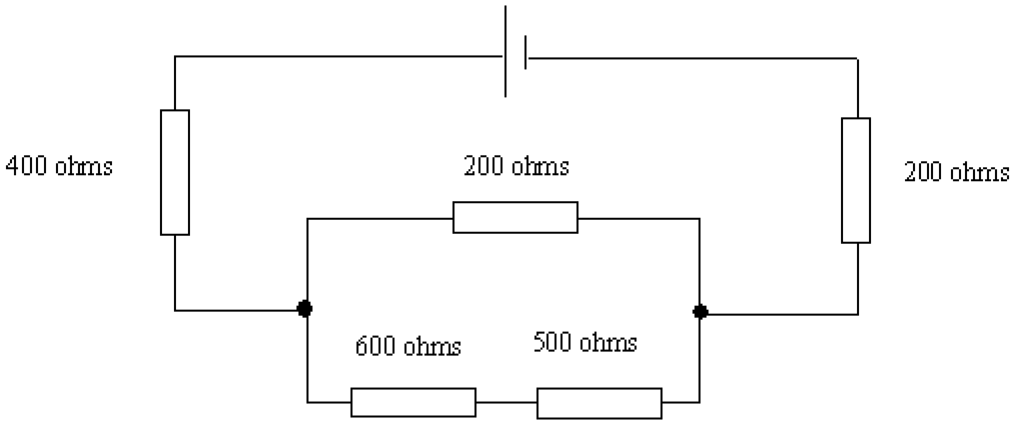
(a) What is the total resistance of the circuit? (Watch out for the bear trap.)

(b) What is the current through each resistor?

(c) What is the total current?

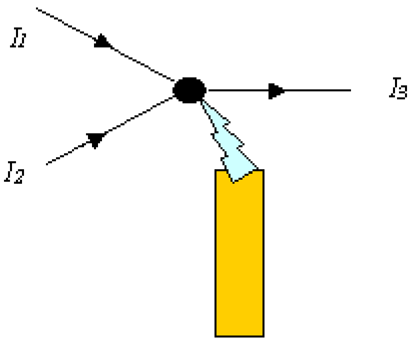
4.06.3

What is the single resistor equivalent of this circuit below?



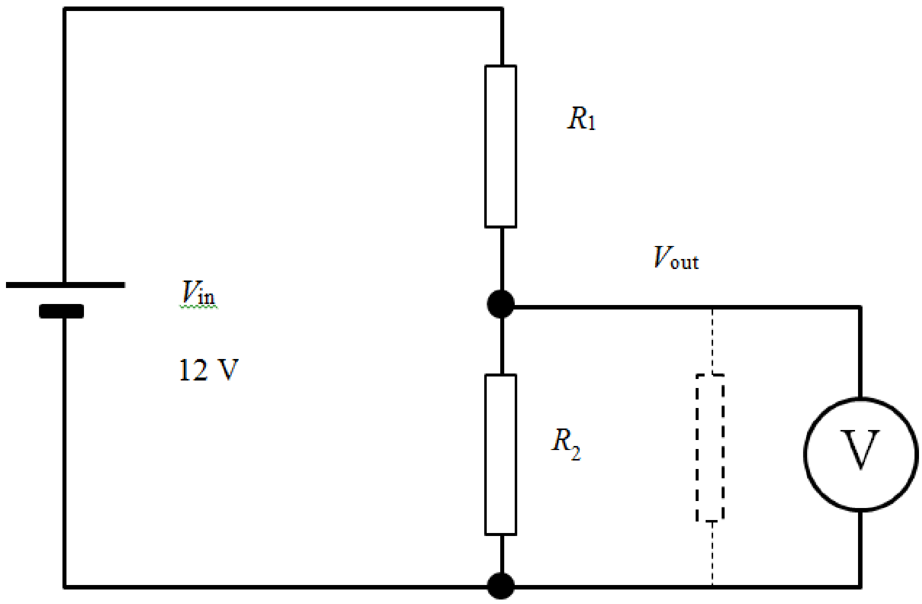
4.06.4

Suppose we had a high voltage junction where was a fault. A spark was jumping as well as current flowing away (i.e. not all the current was in the spark.) How is that consistent with Kirchhoff I? The fault is shown in the diagram:



4.06.6

This is a circuit diagram of a potential divider:



The potential divider is a useful circuit. It is often called the **voltage divider** and is useful to electronic engineers. The voltage divider has a formula for the output voltage:

$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2}\right)$$

a. Show that this formula is consistent with Kirchhoff II.

b. Calculate the output voltage if R_1 = 220 Ω and R_2 = 470 Ω

c. A third resistor of 150 Ω is added in the position shown with the dashed lines. What is the output voltage now?

d. Work out the currents and show that they are consistent with Kirchhoff I.

Tutorial 4.07 EMF and Internal Resistance

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Contents

4.071 Electromotive Force

Batteries (or more strictly speaking **cells**) convert **chemical energy** into **electrical energy**. Generators turn **kinetic energy** into electrical energy. In doing so, they keep the negative terminal with an **excess** of electrons and the positive terminal with a **deficiency** of electrons. A battery does a job of work in pumping the electrons around the circuit. Positive charges do not move.

The early day physicists got it wrong when they said that electric current flows from positive to negative. They didn't know about electrons. When the mistake was discovered, they decided to stick to the positive to negative, so all conventional current flows from positive to negative.

A battery is said to produce **EMF** (electromotive force) which is defined as:

the energy converted into electrical energy when unit charge passes through the source.

This is similar to the definition for potential difference that we saw before, except that it describes the conversion to electrical energy, rather than the conversion from electrical energy. It represents the total energy that can be supplied to a circuit. EMF is a **voltage**.

Note that EMF is **energy per unit charge**, NOT a force, which can lead to confusion. Watch out for this particular bear trap.

I assume that an early physicist got a shock from a high voltage. It gave him a fair belt, so he thought it was a force.

A good working definition of emf is the **open circuit terminal voltage** of the battery, i.e. when there is **zero** current flowing. Although the old text books had a complex method for measuring emf using a metre bridge, nowadays a **digital multimeter** will give you a good reading as it takes a very small current indeed.

The energy supplied to a circuit by a battery is given by:

$$\mathcal{E} = \frac{W}{Q}$$

.... *Equation 47*

Where

- W is the energy in J
- Q is the charge in C
- $\mathcal{E}$, curly E, is the physics code for emf in V

No circuit at all is 100 % efficient. Some energy is dissipated in the wires, or even in the battery itself.

4.072 Internal Resistance

All batteries and generators dissipate heat internally when giving out a current, due to **internal resistance**. A **perfect battery** has **no** internal resistance, but unfortunately there is no such thing as a perfect battery. Nickel-Cadmium and Lead-Acid batteries have very low internal resistance, and we can regard these as almost perfect. These batteries can provide very high currents.

Suppose we connect a cell to a high resistance voltmeter. (A **perfect voltmeter** has **infinite** resistance. A **digital** multimeter has a **very high** resistance, so needs a tiny current; it is almost perfect. An ordinary **moving coil voltmeter** has a **relatively low** resistance, so it takes a small but appreciable current.)

In this circuit (*Figure 54*) the voltmeter reads (very nearly) the emf.

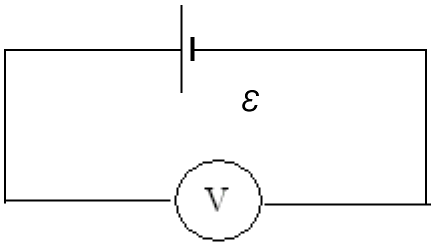


Figure 54 An ordinary voltmeter reads almost the EMF of a cell.

Suppose we now add a load (*Figure 55*). We will assume the wires have **negligible** resistance.

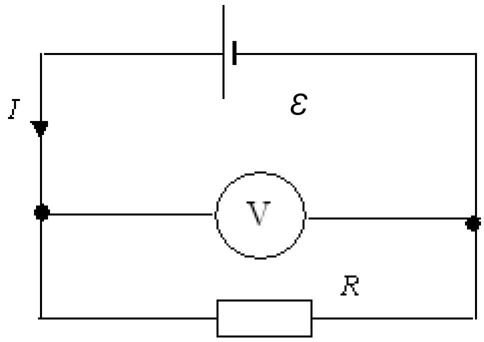


Figure 55 Adding a load to a cell.

This time we find that the terminal voltage goes down to *V*. Since *V* is **less** than $\mathcal{E}$, this tells us that not all of the voltage is being transferred to the outside circuit; some is lost due to the internal resistance which heats the battery up.

Emf = Useful volts + Lost volts

In code:

$$\mathcal{E} = V + v$$ ………… *Equation 48*

So, we can represent the circuit as shown in *Figure 56*.

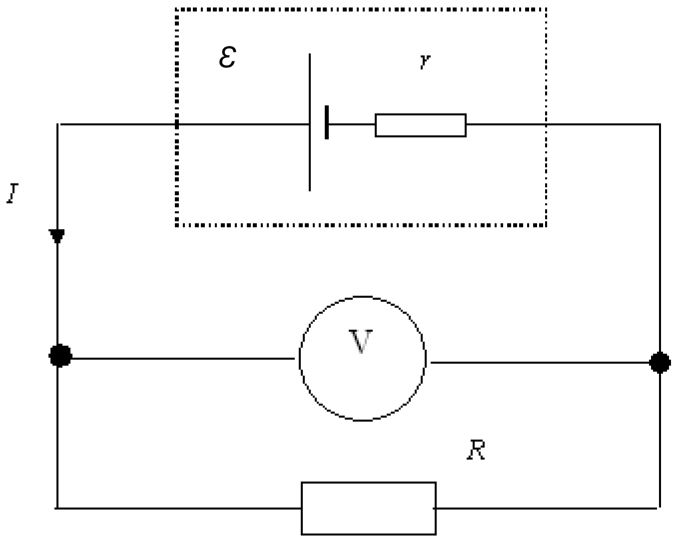


Figure 56 A cell with an internal resistance with an external load.

So, our cell is now a **perfect battery in series with an internal resistor**, r. You cannot open up the battery to find the internal resistor; it is part and parcel of the battery.

We can now treat this as a simple series circuit and we know that the current, I, will be the same throughout the circuit. We also know the voltages in a series circuit add up to the battery voltage.

Emf = voltage across R + voltage across the internal resistance r

$$\mathcal{E} = V + v$$ .... *Equation 49*

We also know from Ohm's Law that $V = IR$ and $v = Ir$. This because the current is the same throughout a series circuit. Therefore, we can write:

$$\mathcal{E} = IR + Ir$$ ....*Equation 50*

Or,

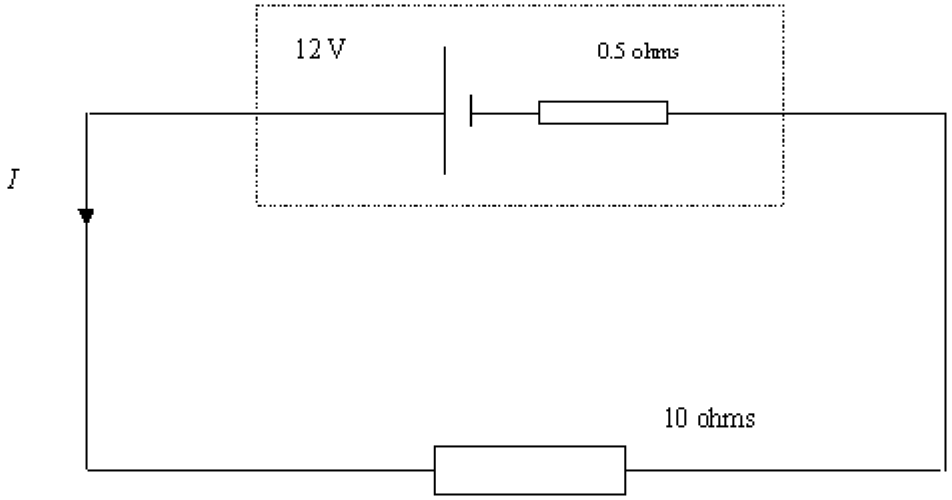
$$\boldsymbol{\mathcal{E} = I\,(R + r)}$$ .... *Equation 51*

Many students panic at the sight of internal resistance problems. I did. All you have to do is turn the cell with the internal resistance into a **perfect battery** in **series** with **its internal resistor** and treat it as a **simple series circuit**.

Let's look at a worked example. It will show you a problem-solving strategy that hopefully will take the anxiety that these problems cause many students.

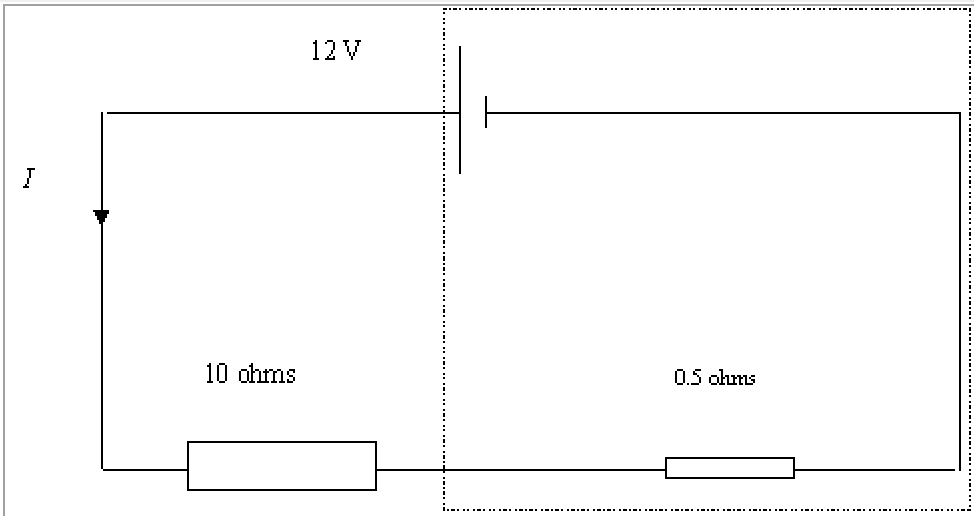
Worked Example

A battery of emf 12 volts and internal resistance 0.5 W is connected to a 10 Ω resistor. What is the current and what is the terminal voltage of the battery under load?



Answer

Step 1: we treat the circuit as a perfect battery in series with an internal resistor. The circuit becomes:



Step 2: Work out the total resistance

$$R_{tot} = R_1 + R_2 = 10\ \Omega + 0.5\ \Omega = 10.5\ \Omega$$

Step 3: Now work out the current:

$$I = V/R = 12\ \text{V} \div 10.5\ \Omega = \underline{1.14\ \text{A}}$$

Step 4: work out the voltage across the internal resistor (lost voltage):

$$v = Ir = 1.14\ \text{A} \times 0.5\ \Omega = 0.57\ \text{volts}$$

Step 5: work out the terminal voltage:

Terminal voltage = emf - lost voltage = 12 V - 0.57 V = **<u>11.43 volts</u>**

We can of course work out the terminal voltage by working the voltage across the 10 Ω resistor, assuming there are no losses.

Here are some important definitions that you will need for the exam.

Term	***Definition***
Cell	Source which converts chemical energy to electrical
Generator	Source which converts kinetic energy to electrical
Emf	Energy per coulomb transferred to electrical energy
Terminal Voltage	Energy per coulomb converted from electrical energy to other forms by the circuit.
Open circuit	Circuit in which zero current is drawn
Internal resistance	Resistance arising from the make-up of the cell due to which a fraction of the electrical energy is lost as heat.

4.073 Measuring Internal Resistance

To measure the internal resistance, we set up the circuit like this (*Figure 57*):

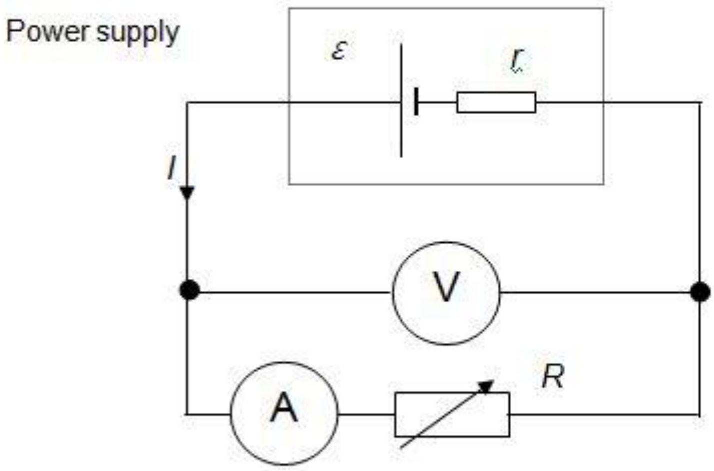


Figure 57 Measuring internal resistance of a cell.

We change the value of the variable resistance. If the resistance is zero, we get a short circuit, so the current will be at the maximum. The voltage will be zero. When the variable resistance is at its highest, the voltage will be less than the emf. We then extrapolate the graph.

In experiments to determine the internal resistance, we get a graph like this (*Figure 58*):

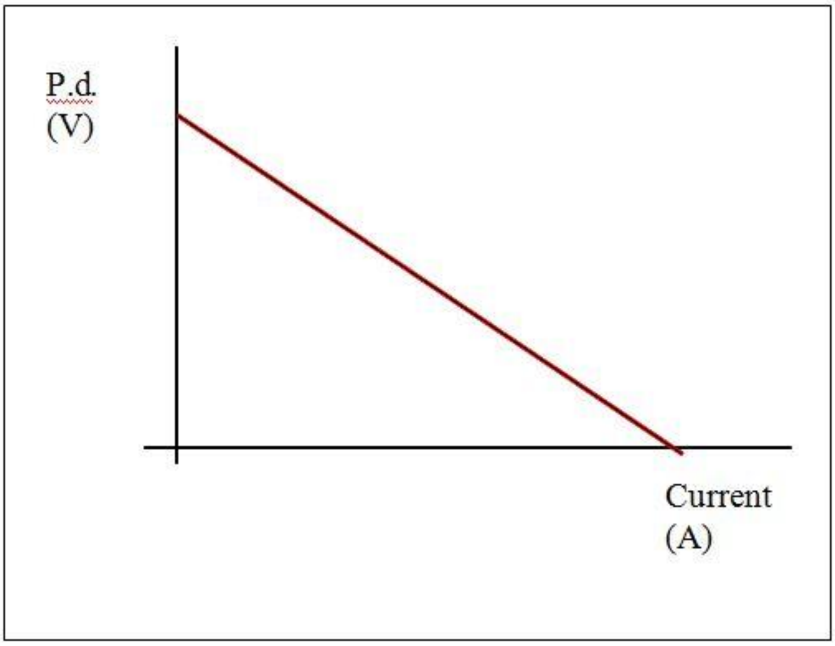


Figure 58 Graph of p.d. against current in a source with internal resistance

The graph is a straight line, of the form $y = mx + c$. We can make the equation for internal resistance $V = -rI + \mathcal{E}$. There are three features on the graph that are useful:

- The **intercept** on the *y*-axis tells as the **emf**.
- The **intercept** on the *x*-axis tells us the **maximum current** the cell can deliver when the p.d. is zero, i.e. a dead short circuit.
- The **negative gradient** tells us the internal resistance.

4.074 Resistance of wires (Extension for A-level Students)

We assume that wires are perfect conductors, i.e. have zero resistance. However, wires have a definite value of resistance. Consider this extension lead commonly found at home:

Figure 59 The conductors of this extension lead have a (very small) internal resistance.

It can carry a current of 13 A. However, it needs to be fully unwound to do so. When carrying a heavy current, the cable gets warm. If it were coiled up, the temperature rise could become excessive, which would be hazardous. There is energy loss in the wires.

The resistances in this wire are:

- 0.4 Ω for the fuses (one fuse in each plug).
- 0.7 Ω for the live wire.
- 0.7 Ω for the neutral wire.

These are actual figures using the extension lead in the picture above, although my ohm meter reads to 0.1 Ω, which represents a 25 % uncertainty. I have done repeats as well, so the data are consistent.

A tumbler drier takes 12 A from the 230 V mains. We can represent the power supply as a perfect source in series with resistances. It is connected to the tumbler drier like this (*Figure 60*):

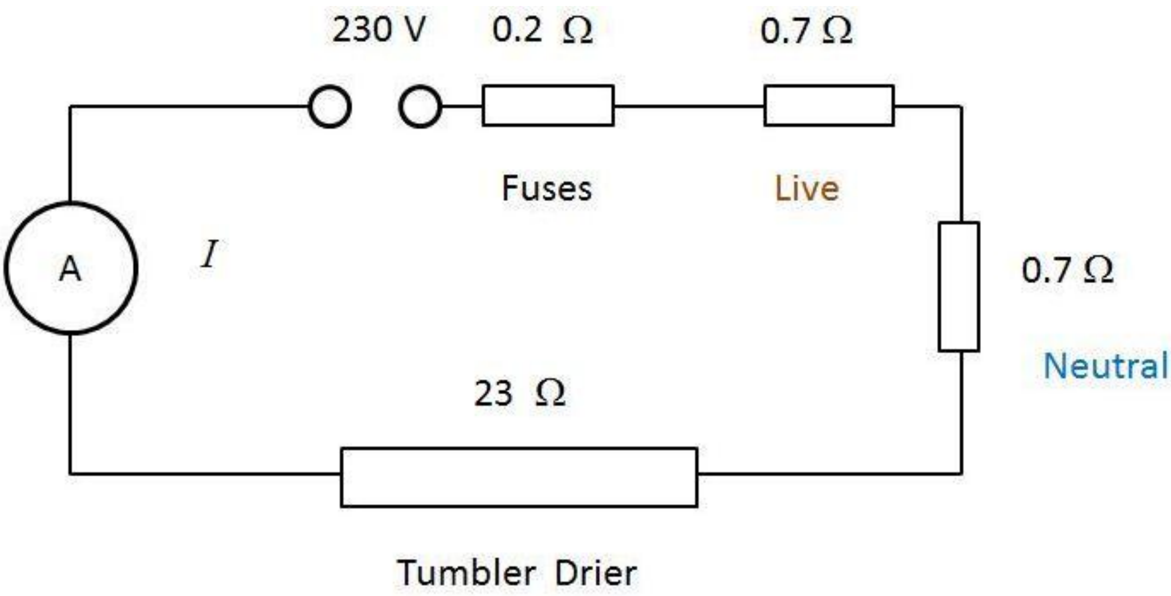


Figure 60 Resistances in the conductors to a high current load.

The cable itself is 15 m long, which results in a power loss of 13 W m^{-1}. The fuse also gets hot.

Tutorial 4.07 Questions

4.07.1

What is the difference between emf and potential difference?

4.07.2

A battery converts 13 000 J of chemical energy into electrical energy. It does so by giving a current of 0.5 A for 2 hours. What is its emf?

4.07.3

What is meant by a perfect battery? Why are real batteries not perfect?

4.07.4

How is this statement consistent with Kirchhoff II?

4.07.5

A battery has an internal resistance of 0.50 Ω. The battery has an emf of 1.52 V. When it is connected to a resistor, the terminal voltage falls to 1.45 V. What current is flowing. What is the value of the resistor?

4.07.6

A tumbler drier of rated power 2300 W is connected to a plug using an extension lead that is 15 m long. The resistance of each conductor is 0.7 Ω and there are two plugs, each of which has a fuse of resistance 0.1 Ω. This is shown in *Figure 60*.

a. Show that the current flowing through the tumbler drier is about 9.4 A.

b. Calculate the voltage across the tumbler drier.

c. Calculate the power across each of the two fuses.

d. Calculate the power lost in the cable.

e. Work out the actual power of the tumbler drier.

Answers to Questions

Tutorial 4.01

4.01.1

An electron can be thought of as a little lump of negative charge.

(Actually, at the level of the electron matter and energy are really one and the same thing. You will have met this idea in Particle Physics)

4.01.2

Charge on the electron is 1.6×10^{-19} C.

No of electrons = 1 ÷ (1.6×10^{-19} C) = **6.2×10^{18} C^{-1}**

4.01.3

Use $Q = It$

$I = Q/t$ = 1.24 C ÷ 0.63 s = **1.97 A**

4.01.4

1 hour = 3600 s

Charge = current × time = 2 A × 3600 s = 7200 C

Energy = charge × voltage = 7200 C × 14.4 V = **103680 J =** 1.04×10^5 J

4.01.5

Opposition to current

Caused by collisions

of electrons with vibrating ions

4.01.6

	V	***I***	***R***
(a)	$V = IR$ = 0.30 A × 18 Ω = **5.4 V**	0.30 A	18 Ω
(b)	12 V	$I = V/R$ = 12 V ÷ 88 Ω **= 0.14 A**	88 W
(c)	14.4	0.52 A	$R = V/I$ = 14.4 V ÷ 0.52 A = **27.7 Ω**

4.01.7 (Extension)

(a) Model the voltmeter as a perfect voltmeter in parallel with a resistor of 35 kΩ.

Work out the resistance of the parallel combination between the 350 Ω and the 35000 Ω resistors:

$$R^{-1} = (350\ \Omega)^{-1} + (35000\ \Omega)^{-1} = 2.89 \times 10^{-3}\ \Omega^{-1}$$

$$R = 346.5\ \Omega$$

Now work out the total resistance:

$$R = 346.5\ \Omega + 0.45\ \Omega + 0.50\ \Omega = 347.485\ \Omega$$

Now work out the current:

$$I = 20\ \text{V} \div 347.485\ \Omega = \mathbf{0.057556\ A} \approx 58\ \text{mA (QED)}$$

(b) Voltage across the voltmeter = 346.5 Ω × 0.057556 A = **19.9 V**

Tutorial 4.02

4.02.1

Voltage and current are directly proportional.

Provided the temperature remains the same.

4.02.2

$R = V/I$ = 12 V ÷ 0.35 A = **34.3 Ω**

4.02.3

$G = I/V$ = 0.35 A ÷ 12 V = **0.029 S**

Tutorial 4.03

4.03.1

The graph is curved,

because the resistance increases (as shown by the gradient).

Filament gets hot.

The resistance goes up,

because the raised temperature causes more vibration of atoms,

leading to increased chances of collisions.

4.03.2

The characteristic graph is not a straight line.

The thermistor gets hot,

therefore, the resistance decreases.

4.03.3

Diode conducts at 0.6 V when forward biased.

But does not conduct when reverse biased.

Until the breakdown voltage,

when it suddenly conducts fully.

Usually, the component is destroyed.

Tutorial 4.04

4.04.1

Work out the radius in metres:

$$r = 0.25 \times 10^{-3}\ \text{m}$$

Now work out the area:

$$A = \pi r^2 = \pi \times (0.25 \times 10^{-3})^2 = \pi \times 6.25 \times 10^{-8}\ \text{m}^2 = 1.96 \times 10^{-7}\ \text{m}^2$$

Now work out l: $R = \rho l / A$

$$10\ \Omega = (47 \times 10^{-8}\ \Omega\text{m} \times l) \div 1.96 \times 10^{-7}\ \text{m}^2$$

$$l = 10\ \Omega \times 1.96 \times 10^{-7}\ \text{m}^2 \div 47 \times 10^{-8}\ \Omega\ \text{m} = \underline{\mathbf{4.18\ m}} = 4.2\ \text{m (2 s.f.)}$$

4.04.2

The resistance is zero at just below the critical temperature.

At the critical temperature, the resistance suddenly rises

from zero to a definite value.

Then the resistance rises slowly with temperature.

4.04.3 (Extension)

Use $I = nAve$

For copper:

Work out the area:

$$A = (\pi \times (1.4 \times 10^{-3}\ \text{m})^2) \div 4 = 1.54 \times 10^{-6}\ \text{m}^2$$

$$v = 0.52\ \text{A} \div (8.0 \times 10^{28}\ \text{m}^{-3} \times 1.54 \times 10^{-6}\ \text{m}^2 \times 1.6 \times 10^{-19}\ \text{C})$$

$$= \mathbf{2.64 \times 10^{-5}\ m\ s^{-1}}\ (= 0.026\ \text{mm/s})$$

For Tungsten:

Work out the area:

$$A = (\pi \times (0.020 \times 10^{-3}\ \text{m})^2) \div 4 = 3.14 \times 10^{-10}\ \text{m}^2$$

$$v = 0.52\ \text{A} \div (3.4 \times 10^{28}\ \text{m}^{-3} \times 3.14 \times 10^{-10}\ \text{m}^2 \times 1.6 \times 10^{-19}\ \text{C})$$

$$= \mathbf{0.304\ m\ s^{-1}}\ (= 300\ \text{mm/s})$$

Tutorial 4.05

4.05.1

Time = 45 min × 60 s min^{-1} = 2700 s

Energy = VIt = 12 V × 3.6 A × 2700 s = **117 000 J**

4.05.2

$I = P/V$ = 60 W ÷ 230 V

= **0.26 A**

4.05.3

$P = I^2R$

1 W = I^2 × 50 W

I^2 = 0.02 A^2

I = √ (0.02 A^2) = **0.14 A**

4.05.4

$P = V^2/R$ = 400 V^2 ÷ 50 Ω = **8 W**

The resistor will fry.

4.05.5

Power = 1000 W.

Time = 3600 s

Energy (J) = power (W) × time (s) = 1000 W × 3600 s = **3.6 × 10^6 J**

You knew that, didn't you?

4.05.6

Time in hours = 1.25 h

Power = 2.3 kW

Cost = 1.25 h × 2.3 kW × 26p (kW h)$^{-1}$ = 74.75 p = **75** p

4.05.7

Units used = 026344 kW h - 025628 kW h = 716 kW h

Cost of electricity used = 716 kW h × 26 p $(kW\ h)^{-1}$ = 18616 p = £186.16

Total bill = £186.16 + £53.00 = **£239.16**

Tutorial 4.06

4.06.1

(a) What is 100 mA in amps?

100 mA = **0.10 A**

(b) What is the current in each resistor?

0.10 A

(c) Work out the voltage across each resistor.

$$V = IR$$

V_1 = 0.10 A × 30 Ω = **3 V**.

V_2 = 0.10 A × 40 Ω = **4 V**.

V_3 = 0.10 A × 50 Ω = **5 V**.

(d) What is the total resistance?

R_{tot} = 30 Ω + 40 Ω + 50 Ω = **120 Ω**

(e) What is the battery voltage?

V = 0.10 A × 120 Ω = **12 V**

4.06.2

(a) Calculate the total resistance of the circuit: (watch out for the bear trap)

$$1/R_{tot} = 1/30\ \Omega + 1/40\ \Omega + 1/50\ \Omega$$

$$= 0.0333\ \Omega^{-1} + 0.025\ \Omega^{-1} + 0.020\ \Omega^{-1} = 0.0783\ \Omega^{-1}$$

$$Rtot = 1/0.0783\ \Omega^{-1} = \mathbf{\underline{12.8\ \Omega}}$$

You didn't fall into the bear trap, did you?

(b) Work out the current through each resistor.

$I = V/R$

$$I_1 = 12\text{ V} \div 30\ \Omega = \mathbf{\underline{0.40\text{ A}}}.$$

$$I_2 = 12\text{ V} \div 40\ \Omega = \mathbf{\underline{0.30\text{ A}}}.$$

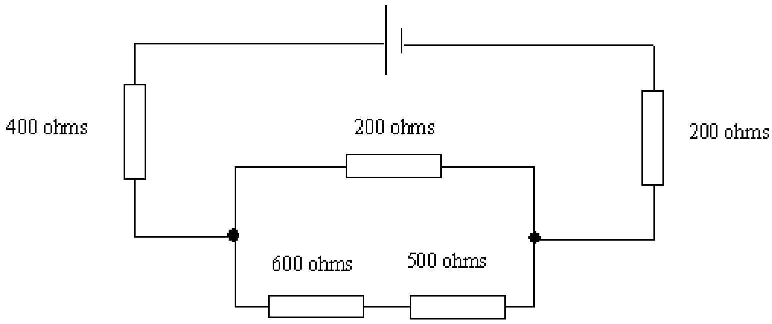
$$I_3 = 12\text{ V} \div 50\text{ W} = \mathbf{\underline{0.24\text{ A.}}}$$

(c) Work out the total current

$$I_{tot} = 0.40\text{ A} + 0.30\text{ A} + 0.24\text{ A} = \mathbf{\underline{0.94\text{ A}}}$$

4.06.3

Here's the circuit again:



First, we will look at the parallel combination:

Add the 500 Ω and the 600 Ω resistors together to get **1100 Ω**.

Now work out the single equivalent resistor for a 200 W and a 1100 W combination:

$$\frac{1}{R_{tot}} = \frac{1}{200\ \Omega} + \frac{1}{1100\ \Omega} = 0.005\ \Omega^{-1} + 9.091 \times 10^{-4}\ \Omega^{-1} = 0.00591\ \Omega^{-1}$$

Note how we have used the decimal reciprocals. Do this if you forget how to add common fractions!

$$R_{tot} = \frac{1}{0.00591\ \Omega^{-1}} = \mathbf{169\ \Omega} = 170\ \Omega \text{ to 2 s.f.}$$

4.06.4

Suppose there were a current I_4 in the spark

The current going out would be $I_3 + I_4$

Which would give the equation $I_1 + I_2 = I_3 + I_4$

4.06.5

a. Kirchhoff II:

$$V_{R1} + V_{R2} + -V_{in} = 0$$

b.

$$V_{out} = 12\ \text{V} \times \frac{(470\ \Omega)}{220\ \Omega + 470\ \Omega} = \mathbf{\underline{8.17\ V}}$$

c. Treat load as two parallel resistors.

$$\frac{1}{R} = \frac{1}{470\ \Omega} + \frac{1}{150\ \Omega}$$

$$R^{-1} = 8.79 \times 10^{-3}\ \Omega^{-1}$$

$$R = \mathbf{\underline{114\ \Omega}}$$

Put this value into the voltage divider equation:

$$V_{out} = 12\ \text{V} \times \frac{114\ \Omega}{220\ \Omega + 114\ \Omega} = \mathbf{\underline{4.10\ V}}$$

d.

The total current = 4.10 V ÷ 114 Ω = 36.0×10^{-3} A

Current in the 470 Ω resistor = 4.10 V ÷ 470 Ω = 8.72×10^{-3} A

Current in the 150 Ω resistor = 4.10 V ÷ 150 Ω = 27.3×10^{-3} A

Total current = 8.72×10^{-3} A + 27.3×10^{-3} A = **$\underline{36.0 \times 10^{-3}\ \text{A}}$**

(which is consistent with Kirchhoff I)

Tutorial 4.07

4.07.1

Emf is the total energy supplied to the circuit per unit charge,

while p.d. is the energy per unit charge converted to other energies by the components.

4.07.2

Convert 2 hours to seconds. 2 h = 7200 s

$Q = It$ = 0.50 × 7200 = 3600 C

Emf = W/Q = 13 000 J ÷ 3600 C = **3.6 V**

4.07.3

A perfect battery has zero internal resistance.

A real battery has internal resistance.

4.07.4

Kirchhoff II says that the voltages in a circuit add up to the emf.

Here we see that there is a p.d. across a resistor

and a p.d. across the internal resistance.

4.07.5

Voltage drop = 1.52 V – 1.45 V = 0.07 V

The current = 0.07 V ÷ 0.5 Ω = 0.14 A

Terminal voltage = 1.45 V

External resistance = 1.45 V ÷ 0.14 A = **10.4 Ω** = 10 Ω to 2 s.f.

4.07.6

a. Total resistance = 23 Ω + 0.2 Ω + 0.7 Ω + 0.7 Ω = 24.6 Ω

The current = 230 V ÷ 24.6 Ω = **<u>9.35 A</u>** = 9.4 A (QED)

b. Voltage = 9.35 A × 23 Ω = **<u>215 V</u>**

c. Power in the two fuses = $I^2 R$ = (9.35 A)^2 × 0.2 W = 17.5 W

There are 2 fuses (one in each plug). Power in each plug = 17.5 W ÷ 2 = **<u>8.75 W</u>**

d. Power in the cable = (9.35 A)^2 × 1.4 Ω = **<u>122 Ω</u>**

e. Power in the tumbler drier = 9.35 A × 215 V = **<u>2010 W</u>**